		GUJARAT TECHNOLOGICAL UNIVERSITY BE- SEMESTER- 1 st / 2 nd • EXAMINATION – SUMMER 2016	
Si Si	ubje ubje	ct Code: 110008 Date:02/06/2016 ct Name:Maths-I	
T	ime:	02:30 PM to 05:30 PM Total Marks: 70	
IN	struci	 Attempt any five questions. Make suitable assumptions wherever necessary. Figures to the right indicate full marks. 	
Q.1	(a)	(i) State Sandwich theorem and using it find $\lim_{x\to 0} g(x)$ if $3-x^2 \le g(x) \le 3 \operatorname{secx}$ for all x.	03
		(ii) Find the point 'c' of Mean Value Theorem for the function $f(x) = 1 - x^2$ in $0 \le x \le 2$.	04
	(b)	(i) Evaluate using L' Hospital rule $\lim_{x\to 0} \frac{x^2 + 2\cos x - 2}{x\sin^3 x}$	03
		(11)Find the Taylor's series expansion of $f(x) = x^3 - 2x + 4$ about a=2	04
Q.2	(a)	(i) Trace the curve $r = a (1 + \cos\theta)$; $a > 0$ (ii) Evolute improper integral $\int_{-\infty}^{\infty} \frac{1}{2} dx$	04 03
	(b)	(ii) Evaluate improper integral $\int_0^{\infty} \frac{1}{x^2} dx$	04
	(0)	(i) Is $\int_4^{\infty} \frac{\sin x}{\sqrt{x(x-1)}} dx$ convergent?	04
		(ii) find the extreme values of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$	03
Q.3	(a)	(i) Check that the sequence $a_n = \frac{n}{n^2 + 1}$ is decreasing and bounded below. Is it convergent?	04
		(ii) Test the convergence $\sum_{n=1}^{\infty} \frac{n^2 + 2}{n^2}$	03
	(b)	Test the convergence of following series:-	07
		(i) $\sum_{n=1}^{\infty} ne^{-n^2}$ (ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$	01
Q.4	(a)	$\left(x\frac{1}{4}+y\frac{1}{4}\right)$	
		(i) If $u = \sin^{-1}\left(\frac{\frac{u}{1}+y}{\frac{1}{x}+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20} tanu$	04
		(ii) If $u = x^2 - y^2$, where $x = 2r - 3s + 4$, $y = -r + 8s - 5$, find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$	03
	(b)	(i) If $u = x + y$ and $v = \frac{x}{x+y}$ find $\frac{\partial(u,v)}{\partial(x,y)}$	03
		(ii) Find the equation of normal line and tangent line at the point (1,1,1) on the surface $x^2 + y^2 + z^2 = 3$.	04
Q.5	(a)	(i) Find the area included between the cardioids $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$.	04
		(ii) Evaluate: $\int_{-\infty}^{4} \int_{-\infty}^{x^2} e^{\frac{y}{x}} dv dx$	03

(ii) Evaluate: J₀ J₀ ex dydx
(b) (i) Use Lagrange method of undetermined multipliers to find the shortest distance from the point (1,2,2) to the sphere x² + y² + z² = 16 07

Q.6	(a)	(i) Evaluate $\int_{0}^{1} \int_{x^2}^{2-x} xy dA$ by changing the order of integration.	04
		(ii) Find the volume of the solid of revolution of the area bounded by the curve $y = xe^x$ and the straight lines x=1, y=0.	03
	(b)	(i) Evaluate: $\int_{0}^{1} \int_{0}^{\sqrt{z}} \int_{0}^{2\pi} (r^2 \cos\theta^2 + z^2) r d\theta dr dz$	03
		(ii) Find the directional derivative of $f(x, y, z) = xyz$ at the point (-1,1,3) in the direction of the vector $\overline{a} = \overline{i} - 2\overline{j} + 2\overline{k}$.	04
Q.7	(a)	(i) Evaluate $\int_0^4 x^3 \sqrt{4x - x^2} dx$ (ii) Find grad(\emptyset) = log($x^2 + y^2 + z^2$) at the point (1,0,-2)	03 04
	(b)	(i) Using Green's Theorem, evaluate the line integral $\int_{c} (siny dx + cosx dy)$ counter clockwise, where C is the boundary of the triangle with vertices (0,0) and	04
		(π ,0) (ii) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is irrotational	03
