Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV EXAMINATION – SUMMER 2016

Subject Code:140001 **Date:26/05/2016**

Subject Name:Mathematics-4

Time:10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Attempt the following. **[07]**

1. State the necessary and sufficient condition for a function to be analytic and prove the necessary condition. **[04]**

2. Obtain Cauchy-Riemann equations in Polar coordinates. **[03]**

(b) Attempt the following. **[07]**

1. Verify that $u = x^2 - y^2 - y$ is harmonic function in entire complex plane and find a harmonic conjugate function *v* of *u*. **[04]**

2. An electrostatic field in *xy*-plane is given by $\phi = 3x^2y - y - y^3$, then find the stream function. **[03]**

Q.2 (a) Attempt the following. **[07]**

1. Define bilinear transformation. Also find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. **[04]**

2. Find the general value of $\log(-3)$. **[03]**

(b) Attempt the following. **[07]**

1. Show that the transformation $w = \dfrac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight the line $4u + 3 = 0$. **[04]**

2. If $f(z)$ is regular function z then show that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **[03]**

OR

(b) Attempt the following. **[07]**

1. Expand $f(z) = -\dfrac{1}{(z-1)(z-2)}$ in the region **[04]**

a) $|z| < 1$ **b)** $1 < |z| < 2$ **c)** $2 < |z| < \infty$

2. Under the transformation $w = \dfrac{1}{z}$, find the image of $|z - 2i| = 2$. **[03]**

Q.3 (a) Attempt the following. **[07]**

1. State and prove Cauchy's integral theorem. **[04]**

2. Evaluate $\oint_C \dfrac{e^{-z}}{z+1} dz$, Where C is the circle **[03]**

(a) $|z| = 2$ (b) $|z| = 1/2$.

(b) Attempt the following. **[07]**

1. Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. **[04]**

2. Use Bisection method to find the real root of the equation $x^3 - x - 11 = 0$ correct up to four decimal places. **[03]**

OR

Q.3 (a) Attempt the following. **[07]**

1. Using Residue theory prove that $\int_0^{\pi} \dfrac{d\theta}{a + b\cos\theta} = \dfrac{\pi}{\sqrt{a^2 - b^2}}$. **[04]**

2. Evaluate the integral $\oint_C \dfrac{4 - 3z}{z^2 - z} dz$ counterclockwise around any closed path such that (a) 0 & 1 inside C (b) 0 is inside & 1 is outside (c) 0 is outside & 1 are inside (d) 0 & 1 both are outside. **[03]**

(b) Attempt the following. **[07]**

1. Use power method to find the largest of Eigen values of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$. Perform four iterations only. **[04]**

2. Compute the real roots of $x \log_{10} x - 1.2$, correct to three decimal place using Newton-Raphson method. **[03]**

Q.4 (a) Attempt the following. **[07]**

1. Compute $\cosh(0.56)$ using Newton's forward difference formula and also estimate the error for the following table. **[04]**

x	0.5	0.6	0.7	0.8
$f(x)$	1.127626	1.185465	1.255169	1.337435

2. Solve the following linear system of equations by Gauss elimination method. **[03]**

$3x + y - z = 3$

$2x - 8y + z = -5$

$x - 2y + 9z = 8$

(b) Attempt the following. **[07]**

1. Evaluate $\int_0^1 e^{-x^2} dx$ by Trapezoidal rule with n=10 and estimate the error. **[04]**

2. Evaluate $\int_0^1 \dfrac{dx}{1+x}$ using Gauss Quadrature of three points. Compare the result with analytic value. **[03]**

OR

Q.4 (a) Attempt the following. **[07]**

1. State Newton's divided difference interpolation formula and compute $f(9.2)$ from the following data. **[04]**

x_j	8.0	9.0	9.5	11.0
$f(x_j)$	2.079442	2.197225	2.251292	2.397895

2. Solve the following linear system of equations by Gauss-Seidel

$$5x + y + 2z = 19$$
$$x + 4y - 2z = -2 \ .$$
$$2x + 3y + 8z = 39$$

(b) Attempt the following. **[07]**

1. Evaluate the integral $\int_1^2 \frac{2x}{1+x^4}dx$, using Gauss-Legendre 3-point quadrature rules. Compare with the exact solution. **[04]**

2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $1/3$ – rule taking $h = \frac{1}{4}$. **[03]**

Q.5 (a) Attempt the following. **[07]**

1. Explain the Euler's method to find Numerical solution of $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$. **[04]**

2. Use Runge–Kutta second order method to find the approximate value of $y(0.2)$ given that $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$ & $h = 0.1$ **[03]**

(b) Attempt the following. **[07]**

Apply Euler's method to the following initial value problem **[04]**

1. $\frac{dy}{dx} = x + y | y(0) = 0$ choosing h=0.2 and obtain y(1).

2. Apply Improved Euler's method to the following initial value problem **[03]** $\frac{dy}{dx} = x + y | y(0) = 0$ choosing h=0.2 and obtain y(1).

OR

Q.5 (a) Attempt the following. **[07]**

1. Derive the Newton Raphson iterative scheme by drawing appropriate figure. **[04]**

2. Apply Runge-kutta fourth order method to the following initial value problem $\frac{dy}{dx} = x + y | y(0) = 0$ choosing h=0.2 and obtain y(1). **[03]**

(b) Attempt the following. **[07]**

Find the Lagrange interpolating polynomial from the following data **[04]**

1.

x	0	1	4	5
$f(x)$	1	3	24	39

2. Prove that $f[x_0, x_1, --, x_n] = \frac{(-1)^n}{x_0 \circ x_1 \circ -- x_n}$. **[03]**

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