Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(New) EXAMINATION – SUMMER 2016

Subject Code:2140001 **Date:26/05/2016**

Subject Name:Mathematics-4

Time:10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Short Questions**	**14**
	1	The complex conjugate of $\frac{i}{1-i}$ is _____	
	2	A mapping which preserves only magnitude is known as _____ mapping.	
	3	IF z = cos θ + i sin θ then sin nθ = _____	
	4	The value of $\int_c \frac{e^z}{(z-3)^2}dz$ where C: $\|z\| = 2$ is _____	
	5	Is the set $\|z-1+2i\| \le 2$ domain?	
	6	Find the principal value of $i^{(1-i)}$.	
	7	Prove $\lim_{z\to 1}\frac{iz}{3} = \frac{i}{3}$ by definition.	
	8	Show that $\sin(\log i^i) = -1$	
	9	Define residue	
	10	While evaluating a definite integral by trepezoidal rule, the accuracy can be increased by taking _____ number of sub-intervals	
	11	The relationship between E and Δ is_____	
	12	The order of convergence in Newton – Raphson method is _____	
	13	Iterative formula for finding the square root of N by Newton – Raphson method is _____	
	14	Putting n = 1 in the Newton-cote's quadrature formula rule obtained is _____	
Q.2	**(a)**	Find all the roots of $(1+i)^{2/3}$.	**03**
	(b)	Show that f(z) = log z is analytic everywhere except at the origin.	**04**
	(c)	Prove that $u = x^2 - y^2$ and $v = -\frac{y}{x^2+y^2}$ are harmonic but $u+iv$ is not regular.	**07**
		OR	
	(c)	Examine the nature of the function f(z) = $\begin{cases} \frac{x^3y(y-ix)}{x^6+y^2}, z \neq 0 \\ 0, z = 0 \end{cases}$ in the region including the origin.	**07**
Q.3	**(a)**	Find analytic function f(z) = u + iv if $v = e^x(x\sin y + y\cos y)$.	**03**
	(b)	Evaluate using Cauchy's integral formula $\int_c \frac{3z^2+z+1}{(z^2-1)(z+3)}dz$ where C is the	**04**

circle $|z| = 2$.

(c) Using contour integration evaluate the real integral $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$. **07**

OR

Q.3 **(a)** If f $(z) = u + iv$ is analytic in domain D then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\text{Re}(f(z)|^2 = 2|f'(z)|^2$ **03**

(b) Determine the linear fractional transformation that maps $z_1 = 0$, $z_2 = 1$, $z_3 = \infty$ onto $w_1 = -1$, $w_2 = -i$, $w_3 = 1$ respectively. **04**

(c) Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the region (i) $|z| < 2$ **07**

(ii) $2 < |z| < 4$ (iii) $|z| > 4$.

Q.4 **(a)** Find the dominant Eigen values of A= $\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$ by power method. **03**

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Trapezoidal rule (ii) Simpson's 1/3 rule taking h = 1. **04**

(c) Prove that the transformation $w = \frac{z}{1-z}$ maps the upper half of the z-plane unto the upper half of the w-plane. What is the image of $|z| = 1$ under this transformation? **07**

OR

Q.4 **(a)** Solve the system of equations by Gauss- Seidal Method **03**

$10x_1 + x_2 + x_3 = 6$

$x_1 + 10x_2 + x_3 = 6$

$x_1 + x_2 + 10x_3 = 6$

(b) Evaluate the integral $\int_0^1 \frac{dt}{1+t}$ by one point, two point and three point Gaussian formula. **04**

(c) The population of the town is given below, estimate the population of the year 1895 and 1930 using suitable interpolation **07**

Year , x	1891	1901	1911	1921	1931
Population (in thousand) f(x)	46	66	81	93	101

Q.5 **(a)** Find up to four decimal places the root of the equation $\sin x = e^{-x}$, using Newtons Rapson's Method starting with $x_0 = 0.6$. **03**

(b) Find the negative root of $x^3 - 7x + 3 = 0$ by bisection method up to two decimal places. **04**

(c) Apply improved Euler method to solve the initial value problem $\frac{dy}{dx} = \log(x + y)$ with $y(1) = 2$ taking h = 0.2 for x = 1.2 and x = 1.4 correct up to four decimal places. **07**

OR

Q.5 **(a)** Express the function $\frac{3x^2 - 12x + 11}{(x-1)(x-2)(x-3)}$ as a sum of partial fraction, using Lagrange's formula. **03**

(b) Using Newton's divided difference formula find a polynomial and also find **04**

f(6).

x	1	2	4	7
f(x)	10	15	67	430

(c) Apply fourth order Runge – Kutta Method to find y (0.2) given $\frac{dy}{dx} = x + y$, y (0) = 1 (Taking h = 0.1). **07**

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