Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 4 (NEW SYLLABUS) EXAMINATION- SUMMER 2016

Subject Code: 2140606 **Date: 26/05/2016**

Subject Name: Numerical and Statistical Methods for Civil Engineering

Time: 10:30 AM to 1:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Answer the following questions in brief:** **14**

1 An unbiased coin is tossed 3 times. What is the probability of obtaining two heads?

2 A 4 sided fair die is thrown twice. What is the probability that the sum of the two outcomes is equal to 6?

3 Write at least two differences between Secant method and False Position method.

4 Which method is also known as the method of Tangents? Write down the iterative formula for it.

5 Define curve fitting.

6 Which method is used to find the dominant Eigen Value.

7 If the value of the coefficient of correlation is negative than what does it signify about the relationship of two variables?

8 Define Mode and also give the relationship between Mean, Median and Mode.

9 Determine the point of intersection of the regression line of y on x and regression line of y on x.

10 Suppose you're taking another multiple choice test in Mathematics. The test consists of 40 questions, each having 5 options. If you guess at all 40 questions, what are the mean and standard deviation of the number of correct answers?

11 State at least two differences between Newton's Divided difference and Newton's Forward Interpolation method.

12 In the Gauss elimination method for solving the system of linear equations, name the matrix which is obtained after triangularization.

13 Determine $f(x,y)$ for solving the following differential equation by Euler's method. $3\frac{dy}{dx}+5y^2=\sin x,\ y(0)=5$.

14 Using Picard's method determine the first approximation y_1 of the initial value problem $\frac{dy}{dx}=x^2+y^2,\ y(0)=0$.

Q.2 **(a)** Use trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five sub intervals. **03**

(b) In a certain assembly plant, three machines, B1, B2, and B3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective? **04**

(c) Solve the following system of equations by Gauss – Seidel method: **07**
$10x - y - z = 13;\ x + 10y + z = 36;\ x + y - 10z = -35$.

OR

(c) Solve the following system of equations by Gauss elimination method: **07**
$x_1 + 2x_2 + 3x_3 = 10;\ 6x_1 + 5x_2 + 2x_3 = 30;\ x_1 + 3x_2 + x_3 = 10.$

Q.3 **(a)** Derive Newton-Raphson's formula for finding the cube root of a positive number N. Hence find $\sqrt[3]{12}$. **03**

(b) From the Taylor's series for $y(x)$, find $y(0.1)$ correct to four decimal places if $y(0.1)$ correct to four decimal places if $y(x)$ satisfies $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$. Also find $y(0.2)$. **04**

(c) Using Milne's Method, solve $\frac{dy}{dx} = 1 + y^2$ with $y(0) = 0,\ y(0.2) = 0.2027,\ y(0.4) = 0.4228,\ y(0.6) = 0.6841,$ obtain $y(0.8)$ and $y(1)$. **07**

OR

Q.3 **(a)** Use secant method to find root of the equation $\cos x - x e^x = 0$ upto four decimal places. **03**

(b) Find by Taylor's series method the value of y at $x = 0.1$ and $x = 0.2$ to four places of decimal from $\frac{dy}{dx} = x^2 - 1$; $y(0) = 1$. **04**

(c) Using Runge – Kutta method to fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$ with a step – size of 0.1. **07**

Q.4 **(a)** Prove the relation: a) $(1+\Delta)(1-\nabla) = 1$ b) $\nabla - \Delta = -\nabla\Delta$ **03**

(b) The speed, v *m/s*, of a car, t seconds after it starts, is shown in table: **04**

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.6	10.08	18.90	21.60	18.54	10.26	5.40	4.50	5.40	9.00

Using Simpson's $1/3^{rd}$ rule, find the distance traveled by the car in 2 minutes.

(c) The compressive strength of samples of cement can be modeled by a normal distribution with a mean 6000 kg/cm^2 and a standard deviation of 100 kg/cm^2. **07**

(i) What is the probability that a sample's strength is less than 6250 kg/cm^2 ?

(ii) What is the probability if sample strength is between 5800 and 5900 kg/cm^2 ?

(iii) What strength is exceeded by 95% of the samples?

[P(z = 2.5) = 0.9938, P(z = 1) = 0.8413, P(z = 2) = 0.9772, P(z = 1.65) = 0.95]

OR

Q.4 **(a)** Prove that: a. $\mu\delta = \frac{1}{2}(\Delta + \nabla)$ b. $\Delta = E\nabla = \nabla E = \delta E^{1/2}$ **03**

(b) Find $\int_0^6 \frac{e^x}{1+x}\,dx$ approximately using Simpson's $3/8^{th}$ rule with h=1. **04**

(c) In a photographic process, the developing time of prints may be looked upon as a random variable having the normal distribution with a mean of 16.28 seconds and a standard deviation of 0.12 second. Find the probability that it will take **07**

(i) anywhere from 16.00 to 16.50 seconds to develop one of the prints;

(ii) at least 16.20 seconds to develop one of the prints;

(iii) at most 16.35 seconds to develop one of the prints.

[P(z = 1.83) = 0.9664, P(z = 0.66) = 0.7454, P(z = 0.58) = 0.7190]

Q.5 **(a)** Find the real root of the equation $x\log_{10} x = 1.2$ by Regula – Falsi method correct to four decimal places. **03**

(b) By the method of least square fit a curve of the form $y = ax^b$ to the following data: **04**

x:	2	3	4	5
y:	27.8	62.1	110	161

(c) Find the value of $\tan 33^0$ by Lagrange's interpolation formula if $\tan 30^0 = 0.5774,\ \tan 32^0 = 0.6249,\ \tan 35^0 = 0.7002,\ \tan 38^0 = 0.7813.$ **07**

OR

Q.5 **(a)** Potholes on a highway can be a serious problem. The past experience suggests that there are, on the average, 2 potholes per mile after a certain amount of usage. It is assumed that the Poisson process applies to the random variable "number of potholes." What is the probability that no more than 4 potholes will occur in a given section of 5 miles? **03**

(b) The pH of a solution is measured eight times by one operator using the same instrument. She obtains the following data: 7.15, 7.20, 7.18, 7.19, 7.21, 7.20, 7.16, and 7.18. Calculate the sample mean, the sample variance and sample standard deviation. **04**

(c) A study of the amount of rainfall and the quantity of air pollution removed produced the following data: **07**

Daily rainfall *x* (0.01 cm)	4.3	4.5	5.9	5.6	6.1	5.2	3.8	2.1	7.5
Particulate removed, $y(\mu g / m^3)$	126	121	116	118	114	118	132	141	108

(i) Find the equation of the regression line to predict the particulate removed from the amount of daily rainfall.

(ii) Find the amount of particulate removed when daily rainfall is x = 4.8 units.

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