Seat No.: ________ Enrollment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER– 1st / 2nd EXAMINATION– SUMMER 2016**

**Subject Code: MTH001** **Date:30/05/2016**

**Subject Name: Calculus**

**Time:02:30 PM to 5:30 PM** **Total Marks: 70**

**Instructions:**

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **MARKS**

**(a)** **Select the correct answer:** **07**

**1.** $f(x)$ is strictly increasing function on $\mathbf{R}$, then ________.

(A) $f'(x)=0,\forall x$ (B) $f'(x)<0,\forall x$ (C) $f'(x)>0,\forall x$ (D) $f''(x)=0,\forall x$

**2.** The slope of the line tangent to the curve $y=2x^2$ at the point where $x=1$ is ________ .

(A) 4 (B) 6 (C) 2 (D) 1

**3.** If $z=x+y$ then $\frac{\partial z}{\partial y}=$ ________.

(A) 1 (B) 2 (C) $x$ (D) $y$

**4.** Equation of tangent plane of $z=x$ at (2,0,2) is _______.

(A) $x+z=0$ (B) $x+y+z=0$ (C) $z=x$ (D) $z=y$

**5.** If the equation of the curve contains only even powers of $x$ and $y$, then the curve is symmetrical about _______.

(A) $x$-axis (B) $y$-axis (C) Both $x$ and y-axis (D) origin

**6.** The curve $r=2a\cos\theta$ represents _______.

(A) cardioid (B) lemniscate (C)Parabola (D) circle

**7.** Area of the circle $x^2+y^2=a^2$ is _______.

(A) $2\pi a$ (B) $2\pi a^2$ (C) $\pi a$ (D) $\pi a^2$

**(b)** **Select the correct answer:** **07**

**1.** The series $\sum\frac{1}{n}$ is ______.

(A) Convergent (B) Divergent (C) Non convergent (D) None of these

**2.** If $u(x,y,z)=0$, then the value of $\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x}$ = ______.

(A) -1 (B) 1 (C) 0 (D) None of these

**3.** Euler's theorem is applicable on _______ function.

(A) Homogeneous (B) Non Homogeneous (C) Constant (D) A and B both

4. If $z = x^2 + y^2$, then $\frac{\partial z}{\partial x} =$ ________.

(A) $2y$ (B) 0 (C) $2z$ (D) $2x$

5. A point $(a,b)$ is said to be local maximum if $(a,b)$ ________.

(A) $rt - s^2 > 0,\ r > 0$ (B) $rt - s^2 < 0$ (C) $rt - s^2 > 0,\ r < 0$ (D) $rt - s^2 = 0,\ r < 0$

6. $\int_1^2 dx =$ ________.

(A) 1 (B) 2 (C) 3 (D) 4

7. The equation of a cylindrical surface $x^2 + y^2 = 16$ becomes ________, when converted to cylindrical polar coordinates.

(A) $r = 16$ (B) $r = 4$ (C) $r = \pm 2$ (D) $r = \pm 4$

**Q.2** **(a)** If $z = e^{xy}, x = t\cos t,\ y = t\sin t$ then find $\frac{dz}{dt}$ at $t = \pi/2$. **03**

**(b)** If resistor of $R_1, R_2$ and $R_3$ ohms are connected in parallel to make on R-ohm resistor, the value of R can be found from the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 10,\ R_2 = 15$ and $R_3 = 30$ ohms. **04**

**(c)** Find extreme values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **07**

**Q.3** **(a)** Evaluate $\iint (x^2 + y^2)\,dx\,dy$, over the region bounded by the lines $y = 4x, x + y = 3, y = 0, y = 2$. **03**

**(b)** Show that the function **04**

$$f(x,y) = \begin{cases} \dfrac{y^2 - x^2}{y^2 + x^2} & ;\quad (x,y) \neq (0,0) \\ 0 & ;\quad (x,y) = (0,0) \end{cases}$$

is not continuous at (0,0).

**(c)** Test the convergence of the series, $\sum_{n=1}^{\infty} \left( \frac{x^n}{n^2 + 1} \right)$, for $x > 0$. **07**

**Q.4** **(a)** Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n - 1}{n(n+1)(n+2)}$ **03**

**(b)** Evaluate $\iint_R (x^2 + y^2)\,dx\,dy$, where R is the region in the $xy$ -plane bounded by $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2 (b > a)$ by changing into polar coordinates. **04**

**(c)** **03**

**(1)** Evaluate $\int_1^3 \int_1^{1/x} \int_0^{\sqrt{xy}} xy\,dz\,dy\,dx$

**(2)** Expand $e^x \cos y$ in the powers of $(x-1)$ and $\left(y-\pi/4\right)$. **04**

**Q.5** **(a)** Express $f(x)=2x^3+3x^2-8x+7$ in terms of $(x-2)$. **03**

**(b)** Find the minimum value of $x^2yz^3$ subject to the condition $2x+y+3z=a$. **04**

**(c)** Test the convergence of the series $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots.$ **07**

**Q.6** **(a)** Evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dA$ by changing the order of integration. **03**

**(b)** Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{2^n}{n^3+1}\right)$. **04**

**(c)** **(1)** If $z=f(u,v)$ and $u=x\cos\theta-y\sin\theta, v=x\sin\theta+y\cos\theta$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=u\frac{\partial z}{\partial u}+v\frac{\partial z}{\partial v}$ **04**

**(2)** If $x^5+y^5=5x^2$ , find $\frac{dy}{dx}$. **03**

**Q.7** **(a)** Evaluate $\iiint x^2yz\,dxdydz$ over the region bounded by the planes $x=0, y=0, z=0$ and $x+y+z=1$ **03**

**(b)** Test the convergence of the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\ldots$ **04**

**(c)** Evaluate $\iiint \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the region bounded by the sphere $x^2+y^2+z^2=a^2$ **07**

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