Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III EXAMINATION – WINTER 2015

Subject Code:130001 **Date:31/12/2015**

Subject Name: Mathematics-III

Time: 2:30pm to 5:30pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1
14

1 Solve $\frac{dy}{dx} = \frac{e^x}{e^y} + x^2 e^{-y}$.

2 Find $L(e^{-3t} t^{3/2})$

3 Find $L^{-1}(\frac{6(s+1)}{s^4})$

4 Find value of $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx$.

5 Determine singular points of differential equation $x(x-1)^3 y'' + 2(x-1)^3 y' + 3y = 0$.

6 $Solve\ (D^4 - 16)y = 0$.

7 Using Bessel's function of the first kind prove that $J_0(0) = 1$.

Q.2 (a) **(1)** Find $L(\frac{e^{-bt} - e^{-at}}{t})$. **02**

(2) Find the Laplace transform of the periodic function **05**

$$f(t) = 3t, 0 < t < 2$$
$$= 6, 2 < t < 4 .$$

(b) **(1)** $Solve\ (D^2 - 5D + 6)y = e^{2x} sin2x$. **04**

(2) Solve $ydx - xdy + logx\, dx = 0$. **03**

OR

(b) **(1)** Find the power series solution about x=0 of $y'' + xy' + x^2 y = 0$. **04**

(2) Find $L^{-1}(\frac{4s+5)}{(s-1)^2 (s+2)})$ **03**

Q.3 (a) **(1)** Solve $x^2 \frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + 6y = xlogx$. **04**

(2) Solve $\frac{y^2}{x} dx + (1 + 2ylogx)dy = 0$ **03**

(b) **(1)** Using method of undetermined co-efficients solve the differential equation $y'' + 9y = 7e^{3x} + 3x^2 + 4, y(0) = 0, y'(0) = 1$. **04**

(2) Find Orthogonal trajectories of $r^n = a^n\, cosn\theta$. **03**

OR

Q.3 **(a)** **(1)** Using Laplace transform solve **04**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}sint, x(0) = 0, x'(0) = 1$$

(2) Using convolution theorem find $L^{-1}(\frac{2}{(s^2+1)(s^2+4)})$. **03**

(b) **(1)** Express the following in terms of unit step function **04**

$$f(t) = t - 1, 1 < t < 2$$
$$= 3 - t, 2 < t < 3$$

(2) Find $L(te^{-t}cosht)$ **03**

Q.4 **(a)** Find the general solution of $2x^2y'' + xy' + (x^2 - 1)y = 0$ by using Frobenius method. **07**

(b) **(1)** Show that $x^4 = \frac{1}{35}[8P_4(x) + 20P_2(x) + 7P_0(x)]$ **04**

(2) Show that $\int_{-1}^{1} P_m(x).P_n(x)\,dx = 0\ if\ m \neq n.$ **03**

OR

Q.4 **(a)** Find series solution of the differential equation $y'' + xy = 0$ **07**

(b) **(1)** State Legendre duplication formula and hence prove that **04**

$\beta(m,m).\beta\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \pi m^{-1}.2^{1-4m}$

(2) Evaluate $\int_3^7 \sqrt[4]{(x-3)(7-x)}\,dx$ **03**

Q.5 **(a)** Expand f(x) as a fourier series in the interval $(0, 2\pi)$ if **07**

$$f(x) = -\pi, 0 < x < \pi$$
$$= x - \pi, \pi < x < 2\pi$$ hence show that

$$\sum_{n=1}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8.}$$

(b) Express the function $f(x) = -e^{kx}, x < 0$ **07**

$= e^{-kx}, x > 0$ as fourier integral and

hence show that $\int_0^{\infty} \frac{wsinwx}{w^2+k^2}dw = \frac{\pi}{2}.e^{-kx}\ if\ x > 0, k > 0$.

OR

Q.5 **(a)** Solve by the method of separation of variables $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ **07**

(b) Find half range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$ and deduce that **07**

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6.}$$

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