Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III EXAMINATION – WINTER 2015

Subject Code:130002 **Date:31/12/2015**

Subject Name: Advanced Engineering Mathematics

Time: 2:30pm to 5:30pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	Attempt the following	
	(i)	Solve $\frac{dy}{dx}+2y\tan x=\sin x.$	**4**
	(ii)	Solve $(e^y+1)\cos x dx+e^y\sin x dy=0.$	**3**
	(b)	Attempt the following	
	(i)	Solve $(D^2-5D+6)y=x+e^{4x}.$	**4**
	(ii)	Define triangular wave function and draw its graph.	**3**
Q.2	**(a)**	Attempt the following	
	(i)	Solve $(D^2-8D+9)y=40\sin 5x.$	**4**
	(ii)	Find ordinary and singular points for $2x(x-2)^2y''+3xy'+(x-2)y=0.$	**3**
	(b)	Attempt the following	
	(i)	Solve $(1+x)^2y''+(1+x)y'+y=4\cos\log(1+x).$	**5**
	(ii)	Define beta function. Find *B(5,4).*	**2**
		OR	
	(b)	Find the power series solution about *x=0* of $y''+xy'+x^2y=0.$	**07**
Q.3	**(a)**	Express $f(x)=x\cos x$ as a Fourier series in $(-\pi,\pi).$	**07**
	(b)	Find Fourier series expansion of the function given by $f(x)=\begin{cases}0, -2<x<0\\ 1, 0<x<2\end{cases}$	**07**
		OR	
Q.3	**(a)**	If $f(x)=\begin{matrix}x, 0\le x\le\pi\\ 2\pi-x, \pi\le x\le 2\pi\end{matrix}$, find the Fourier series.	**07**
	(b)	Prove that $\frac{l}{2}-x=\frac{l}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{2n\pi x}{l}, 0<x<l.$	**07**
Q.4	**(a)**	Attempt the following	
	(i)	Evaluate $L^{-1}\left\{\log\left(\frac{s+4}{s+3}\right)\right\}.$	**4**
	(ii)	Find $L\{t^2\sin 4t\}.$	**3**
	(b)	Attempt the following	
	(i)	Find the Laplace transform of the periodic function defined by $f(t)=\frac{t}{2}, 0<t<3, \quad f(t+3)=f(t).$	**4**

	(ii)	Find Laplace inverse of $\dfrac{1}{s(s+a)^3}$.	**3**

OR

Q.4	**(a)**	Attempt the following	
	(i)	Use convolution theorem to find $L^{-1}\left\{\dfrac{1}{s(s^2+a^2)}\right\}$.	**4**
	(ii)	Find $L\left\{\dfrac{1-\cos 2t}{t}\right\}$.	**3**
	(b)	Attempt the following	
	(i)	Solve by Laplace transform: $\dfrac{dy}{dt}-2y=4$, given that $t=0, y=1$.	**5**
	(ii)	Find $L(t^2 * \cos t)$.	**2**
Q.5	**(a)**	Attempt the following	
	(i)	Derive partial differential equation by eliminating a and b from $z=ax+by+ab$.	**3**
	(ii)	Find the complete integral of $q=pq+p^2$.	**4**
	(b)	Solve the p.d.e. $u_{xx}=16u_y$ by using separation of variables method.	**07**

OR

Q.5	**(a)**	Attempt the following	
	(i)	Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z, x^2+y^2-z^2)=0$.	**3**
	(ii)	Solve $2r+5s+2t=0$.	**4**
	(b)	Using Fourier integral show that $\displaystyle\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda\, d\lambda = \begin{cases} \pi/2, & 0<x<\pi \\ 0, & x>\pi \end{cases}$	**07**

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