

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV EXAMINATION – WINTER 2015****Subject Code: 140001****Date: 19/12/2015****Subject Name: Mathematics-IV****Time: 02:30pm to 05:30pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** (i) Find real and imaginary parts of $(\sqrt{i})^{\sqrt{i}}$ **03**
- (ii) Prove that $\operatorname{sech}^{-1} x = \log \left[\frac{1 + \sqrt{1 - x^2}}{x} \right]$ **04**
- (b)** (i) Define Harmonic function. Show that the function $u(x, y) = e^x \cos y$ is harmonic. Determine its harmonic conjugate $v(x, y)$ and the analytic function $f(z) = u + iv$ **03**
- (ii) State the necessary condition for $f(z)$ to be analytic. For what values of z is the function $f(z) = 3x^2 + i y^2$ analytic? **04**
- Q.2 (a)** (i) Sketch the following sets and determine the sets is open, closed, bounded **04**
- (1) $|2z + 1 + i| < 4$
- (2) $\operatorname{Re} z \geq 4$
- (ii) Define mobius transformation. Determine the mobius transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. **03**
- (b)** (i) Evaluate $\int_C \operatorname{Re}(z^2) dz$, where C is the boundary of the square with vertices $0, i, 1 + i, 1$ in the clockwise direction **04**
- (ii) Determine residue at poles $\left(\frac{2z + 1}{z^2 - z - 2} \right)$ **03**
- OR**
- (b)** (i) Let $f(z) = z^n = r^n e^{i\theta}$ for integer n verify C-R equations and find its derivative. **04**
- (ii) Find the roots common to equations $z^4 + 1 = 0$ and $z^6 - i = 0$ **03**
- Q.3 (a)** State and prove cauchy's Integral formula. **07**
- (b)** Evaluate $\int_C \frac{z}{z^2 + 1} dz$, where C is the circle **07**
- (i) $|z + \frac{1}{2}| = 2$ (ii) $|z + i| = 1$
- OR**
- Q.3 (a)** Using contour Integration show that $\int_0^\infty \frac{dx}{1 + x^4} = \frac{\pi}{2\sqrt{2}}$ **07**

- (b) Expand $\frac{1}{z(z^2 - 3z + 2)}$ in a Laurent series about $z=0$ for the regions 07
 (i) $0 < |z| < 1$
 (ii) $|z| > 2$

- Q.4** (a) Using Newton – Raphson method find a root of the equation $xe^x = 2$ 07
 Correct to three decimal places.

- (b) Using Langranges interpolation formula obtain the value of x for $y=85$ 07

x	2	5	8	14
y	94.8	97.9	81.3	68.7

OR

- Q.4** (a) Use secant method to find the roots of $\cos x - xe^x = 0$ correct 07

upto 3 decimal places of decimal.

- (b) Compute $f(9.2)$ from the following value using Newton's divided difference formula. 07

x	8.0	9.0	9.5	11.10
$F(x)$	2.079442	2.197225	2.251292	2.397895

- Q.5** (a) Evaluate $\int_0^\pi \sin x \, dx$ Take $n=10$ (i) Trapezodial Rule (ii) Simpson's $\frac{1}{3}$ rule 07

- (b) Use power method to find the largest eigen value and corresponding eigen 07

vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 3 \\ 0 & 0 & .0 \end{bmatrix}$

OR

- Q.5** (a) State simpson's $\frac{3}{8}$ Rule and evaluate $\int_0^1 \frac{dx}{1+x^2}$ taking $h = \frac{1}{6}$ 07

- (b) Solve the following linear system of equations by Gauss- sidel method 07

$$\begin{aligned} 10x + y + z &= 12 \\ 2x + 10y + z &= 13 \\ 2x + 2y + 10z &= 14 \end{aligned}$$
