Enrolment No.\_\_\_\_\_

## **GUJARAT TECHNOLOGICAL UNIVERSITY** BE - SEMESTER–IV (New) EXAMINATION – WINTER 2015

0						Date:19/12/2015	ate:19/12/2015	
	Subject Name: Mathematics - IV Time: 2:30pm to 5:30pm Instructions:			v	Total Marks: 70			
		1. 2.	Attempt all o Make suitab	_	vherever necessa full marks.	ry.		
Q.1	<b>(a)</b>			-	$Imz^3$ and $(Imz)$			03 04
	<b>(b)</b>	(ii) Find and graph all roots of $\sqrt[4]{-1}$ in the complex plane. (i) Represent $z = \frac{1+i}{1-i}$ in polar form.						
	(i) Is the function $u = x^3 - 3xy^2$ harmonic? If your ans corresponding analytic function.				nswer is yes, find a	04		
Q.2	(a)	(i) Evaluate $\oint_C \frac{dz}{z^2+1}$ , where C is the circle $ z-i  = 3/2$ .						03
		(ii) Using Cauchy's integral formula integrate $\oint_C \frac{e^z}{z-2i} dz$ , where C is circle $ z-2i  = 4$ .						04
	<b>(b)</b>		(i) Find the	Laurent series of	of $z^2 e^{1/z}$ with co			03 04
		(ii) Determine and sketch the image of $ z  = 1$ under the transformation $w = z + i$ .						
					OR			
	<b>(b)</b>	(i) Find the Taylor's series of $f(z) = \frac{1}{1+z^2}$ at $z_0 = 0$ .						03
		(ii) Evaluate $\oint_C \frac{4-3z}{z^2-z} dz$ , where $C:  z  = 2$ .						04
Q.3	(a)		(i) Evaluate	$\oint_C \frac{e^{2z}}{z-2} dz$ , where	ere C is a circle	z  = 1.		03
			(ii) Using re	sidue theorem e	evaluate $\oint_C \frac{z^2 \sin z}{4z^2 - z}$	$\frac{z}{1}dz$ , where C:  z	z  = 2.	04
	<b>(b)</b>		(i) Evaluate		0 42 -	L		03
		(ii) Find the bilinear transformation which maps the points $z_1 = -1$ , $z_2 =$						
		$0, z_3 = 1 \text{ onto } w_1 = 0, w_2 = 1, w_3 = -1.$ OR						
Q.3	(a)	-						03
					evaluate $\oint_C \frac{30z^2}{(2z-1)}$		2	04
	<b>(b)</b>				- (== -	(02 1)		03
		(i) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $ z  < 1$ . (ii) Determine the bilinear transformation that maps the points $z_1 = 0$ , $z_2 = 0$						
		$1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ .						
Q.4	(a)	Ev	valuate $\int_0^6 \frac{1}{1+x}$	dx taking $h = 2$	1 by using Simps	son's 1/3 rule.		04
	<b>(b)</b>	Determine the interpolation polynomial of degree three using Langrange's 05					05	
		int	$\frac{x}{x}$	the following c -1	lata.	1	3	
		f(x) 2 1 0 -1						

Apply Newton's divided difference formula compute f(0.8) from the data (c)

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x	0.5	1.0	2.0	
f(x)	0.479	0.841	0.909	
OR				

(a) Solve the following linear system of equations by Gauss elimination method. 0.4  $x_1 + x_2 + x_3 = 9$ ;  $2x_1 - 3x_2 + 4x_3 = 13$ ;  $3x_1 + 4x_2 + 5x_3 = 40$ 

- Apply Newton-Raphson method to find the solution of  $f(x) = x^3 5x + 3$  correct 05 **(b)** up to four decimal places.
- (c) Using Euler's method find y(0.1) and y(0.2) given  $\frac{dy}{dx} = y \frac{2x}{y}$ ; y(0) = 1, 05 (Take h = 0.1)
- 0.5 (a) Find the positive solution of f(x) = x - 2sinx = 0 by the secant method correct up 04 to three decimal places. Start from  $x_0 = 2$  and  $x_1 = 1.9$ .
  - Using Rung-Kutta method of fourth order, find y(0.2) given that  $y' = xy + y^2$ , **(b)** 05 v(0) = 1.
  - (c) Apply the Gauss-Seidel iteration (3 steps) to the system (start from (0,0,0)). 05  $10x_1 + x_2 + x_3 = 6$ ;  $x_1 + 10x_2 + x_3 = 6$ ;  $x_1 + x_2 + 10x_3 = 6$ OR

(a) Evaluate  $\int_0^1 e^{-x^2} dx$  by Trapezoidal rule. (Take h=0.1) 0.5

<b>(b</b> )	Using Newton's interpolation formula compute cosh 0.56 from the data,						
	x	0.5	0.6	0.7	0.8		
	coshx	1.127626	1.185465	1.255169	1.337435		

(c) Using Taylor's series method to determine the value of y(0.03) correct up to four 05 decimal places, if  $y' = x^2y - 1$ ; y(0) = 1.

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