Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–IV (New) EXAMINATION – WINTER 2015**

**Subject Code:2140505 Date:19/12/2015**

**Subject Name: Chemical Engineering Maths**

**Time: 2:30pm to 5:30pm Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Define following: **07**

i) Truncation error ii) Round off error iii) Absolute Error iv) Accuracy

v) Error propagation vi) Initial value and boundary value problem

vii) Precision

**(b)** i. Use Descartes' rule of signs to find the number of positive, negative and imaginary roots of the function: $x^6 - x^5 - 10x + 7 = 0$ **03**

ii. Write an algorithm for False Position method **04**

**Q.2** **(a)** i.) Discuss bracketing methods & open methods **03**

ii.) You are designing a spherical tank (Figure - 1) to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as **04**

$$V = 2\pi h^2 \frac{[3R - h]}{3}$$

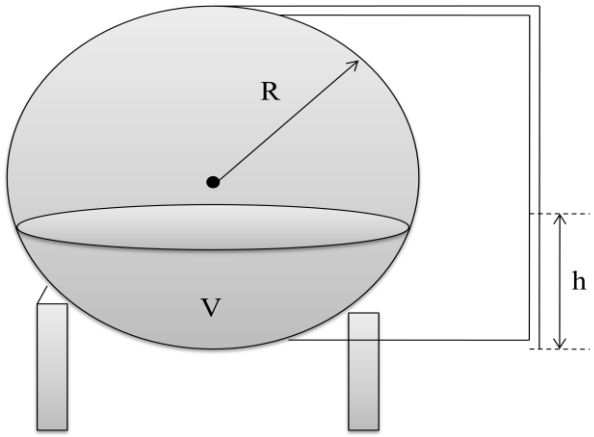


Figure - 1

Where V = volume [$m^3$], h = depth of water in tank [m], and R = tank radius [m]

If R = 3 m, to what depth must the tank be filled so that it holds 30 $m^3$. Use three iterations of bisection method to determine your answer.

[Hint: from the physics of the problem, the depth (h) would be between h = 0 and h = 2R and hence the this becomes the lower and upper limits of depths to initiate the computation]

**(b)** Use the multiple equation Newton - Raphson method to determine roots of the equations: **07**

$u(x, y) = x^2 + xy - 10 = 0$ & $v(x, y) = y + 3xy^2 - 57 = 0$

Initiate the computation with guesses of $x = 1.5$ & $y = 3.5$

**OR**

**(b)** Derive the formula of Newton - Raphson method & also prove that Newton - Raphson method is quadratically convergent. **07**

**Q.3** **(a)** Discuss the pitfalls of Gauss - Elimination method and techniques for improving solutions **07**

**(b)** Define diagonally dominant system. Check whether the following system of equations is diagonally dominant or not? Solve the system by using Gauss - Seidel iterative method (3 iterations). **07**

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

**OR**

**Q.3** **(a)** Derive normal equation to fit a 2nd order polynomial using least square method. **07**

**(b)** In an experiment the following values of heat capacity C at various temperatures T for a gas is obtained. **07**

| Temp (T): | -50 | -30 | 0 | 60 | 90 | 110 |
|---|---|---|---|---|---|---|
| Heat Capacity (C): | 1270 | 1280 | 1350 | 1480 | 1580 | 1700 |

Use linear regression to determine the model to predict heat capacity (C) as a function of temperature (T)

**Q.4** **(a)** The following table gives the values of density (d) of a saturated water for various temperatures (T) of saturated steam **07**

| Temp (T) °C : | 100 | 150 | 200 | 250 | 300 |
|---|---|---|---|---|---|
| Density (d) kg/m$^3$ : | 958 | 917 | 586 | 799 | 712 |

Use Newton's forward interpolation formula to find the density when the temperature is 130 °C and 270 °C.

**(b)** i.) Prove the following: **03**

$E = 1 + \Delta \quad and \quad E = e^{h\Delta}$

ii.) Determine the interpolating polynomial of degree three using Lagrange's interpolation formula for the table below **04**

| x : | -1 | 0 | 1 | 3 |
|---|---|---|---|---|
| F(x) : | 2 | 1 | 0 | -1 |

**OR**

**Q.4** **(a)** i.) Derive formula for trapezoidal rule of numerical integration **04**

ii.) Integration provides a means to compute how much mass enters or leaves a reactor over a specified time period as $M = \int_{t_1}^{t_2} Qc\, dt$ **03**

Where $t_1$ and $t_2$ = the initial and final times respectively.

The integral represents the summation of the product of flow times concentration to give the total mass entering or leaving from time $t_1$ to $t_2$. For a constant flow rate of Q = 4 m$^3$/min, Use Simpson's rule to evaluate this equation for data listed below:

| t (mins) : | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| C (mg/m$^3$): | 10 | 35 | 55 | 52 | 40 | 37 | 32 |

**(b)** The following data gives pressure and volume of a super heated steam **07**

| Volume (V) : | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| Pressure (P) : | 105 | 42.7 | 25.3 | 16.7 | 13 |

Find the rate of change of pressure with respect to volume when V = 2 and V = 8

**Q.5** **(a)** Solve the following initial value problem **07**

$$\frac{dy}{dx} = 4e^{0.8x} - 0.5y\,; \quad y(0) = 2$$

From x = 0 to 0.5 taking h = 0.5 using 4th order Runge - Kutta method.

**(b)** Solve $y' = y + x^2$ with y(0) = 1 using Milne's predictor - corrector method & find y(0.8) taking h = 0.2 with values of y(0.2), y(0.4), y(0.6) listed below **07**

| x : | 0 | 0.2 | 0.4 | 0.6 |
|---|---|---|---|---|
| y : | 1 | 1.2242 | 1.5155 | 1.9063 |

**OR**

**Q.5** **(a)** A steady state heat balance for a rod can be represented as $\frac{d^2T}{dx^2} - 0.15T = 0$ **07**

With $T(0) = 240$ & $T(10) = 150$, Use the finite difference approach with Δx = 2 to solve the problem

**(b)** Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh (Figure - 2) with boundary values as shown. (Use Gauss Seidel method upto three iterations) **07**

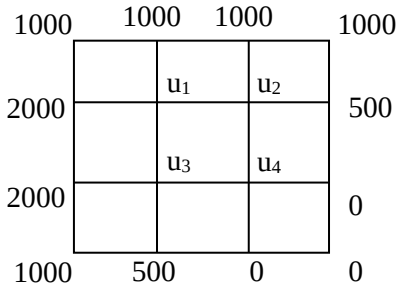


Figure - 2

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