Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (New) EXAMINATION – WINTER 2015

Subject Code:2141005 **Date:04/01/2016**

Subject Name: Signals and Systems

Time: 2:30pm to 5:00pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define : Signal. **07**

Find the fundamental periods (T for continuous-time signals, N for discrete-time signals) of the following periodic signals.

1. $x(t) = \cos(13\pi t) + 2\sin(4\pi t)$
2. $x[n] = e^{j7.351\pi n}$

(b) Define: System. **07**

Determine whether the system $y(t) = t\,x(t)$ is

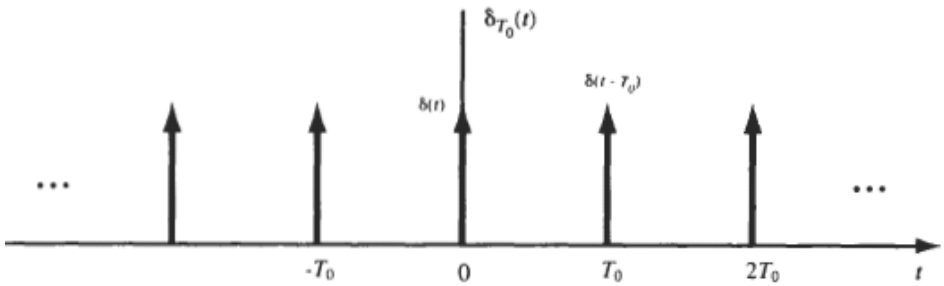
1. Memoryless
2. Linear
3. Time invariant
4. Causal
5. BIBO stable. Justify your answers.

Q.2 **(a)** Compute the convolution y(n) = x(n) * h(n) **07**

1. $x[n] = \delta[n] - \delta[n-2]$, $h[n] = u[n]$
2. $x[n] = u[n]$, $h[n] = u[n]$

(b) Determine the trigonometric Fourier series for signal given below **07**

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0).$$



OR

(b) Determine the complex exponential Fourier series for **07**

1. $\cos(\omega_0 t)$
2. $\sin^2 t$

Q.3 **(a)** Define: The continuous time Fourier transform. **07**

State and prove Time shifting and Duality properties of continuous time Fourier transform.

(b) Find the Z transform of **07**

1. $\delta(\mathrm{n})$
2. $u[\mathrm{n}]$
3. $na^n u[\mathrm{n}]$. (1+2+4 Marks)

OR

Q.3 **(a)** Define: The Z transform. **07**

State and prove Time shifting and Time reversal properties of Z transform.

(b) Find the continuous time Fourier transform

1. $\delta(t)$ **07**
2. $e^{-at}u[\mathrm{t}], \mathrm{a} > \mathrm{o}$
3. $u[\mathrm{t}]$. (1+2+4 Marks)

Q.4 **(a)** Using power series expansion technique find the inverse Z transform of **07**

$\mathrm{X}(z) = \dfrac{1}{1 - az^{-1}}, |z| > |a|$.

(b) The output $y[n]$ of a discrete-time LTI system is found to be $2\left(\dfrac{1}{3}\right)^n u[n]$ when the input $\mathrm{x[n]}$ is $u[n]$.Find the impulse response $\mathrm{h[n]}$ of the system. **07**

OR

Q.4 **(a)** Using the partial fraction expansion technique find the inverse Z transform of **07**

$\mathrm{X(Z)} = \dfrac{z}{2z^2 - 3z + 1}, \ |z| < \dfrac{1}{2}$.

(b) For the differential equation $y[n] - \dfrac{1}{2}y[n-1] = x[n]$ with input $x[n] = \left(\dfrac{1}{3}\right)^n$ and for initially $y[-1] = 1$ find the output $y[n]$. **07**

Q.5 **(a)** Define: Convolution Sum. **07**

Show that

1. $x[\mathrm{n}] * \delta[\mathrm{n}] = x[n]$
2. $x[\mathrm{n}] * \delta[\mathrm{n} - \mathrm{n}_0] = x[n - n_0]$
3. $x[\mathrm{n}] * u[\mathrm{n} - \mathrm{n}_0] = \sum_{k=-\infty}^{n-n_0} x[\mathrm{k}]$

(b) A Continuous-time periodic signal *x(t)* is real valued and has a fundamental period T = 8. The nonzero Fourier series coefficients for *x(t)* are **07**

$$a_1 = a_{-1} = 2, a_3 = a_{-3}^* = 4j.$$

Express *x(t)* in the form

$$x(\mathrm{t}) = \sum_{k=0}^{\infty} A_k \cos(\omega_k \mathrm{t} + \phi_k).$$

OR

Q.5 **(a)** Define the condition for LTI system to be stable. Which of the following impulse responses correspond to stable LTI systems. **07**

1. $h_1(t) = e^{-(1-2j)t}u(t)$
2. $h_2(\mathrm{n}) = 3^n u[-n+10]$

(b) Define Laplace transform. Prove linearity property for Laplace transform. State how ROC of Laplace transform is useful in defining stability of systems. **07**

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