Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (New) EXAMINATION – WINTER 2015

Subject Code:2141905 Date:19/12/2015

Subject Name: Complex Variables and Numerical Methods

Time: 2:30pm to 5:30pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	A	(i)	Sketch the region $1 < \|z+i\| \le 2$ and check whether the region is domain or not?	03
		(ii)	Find all the roots of the equation $\log z = \frac{i\pi}{2}$.	02
		(iii)	Evaluate $\int_C \frac{dz}{z^2}$,C is along a unit circle.	02
	b	(i)	Obtain the Taylor's series $f(z) = \sin z$ in power of $(z - \frac{\pi}{4})$	03
		(ii)	Find the Laurent's expansion of $\frac{\sin z}{z^3}$ at $z = 0$ and classify the singular point z=0.	02
		(iii)	Is $f(z) = \sqrt{r} e^{\frac{i\theta}{2}}$ analytic? $(r > 0 \;, -\pi < \theta < \pi)$	02
Q.2	a		Show that for the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z} \; ; \; z \ne 0 \\ 0 \;\; ; \; z = 0 \end{cases}$ Is not differentiable at $z = 0$ even though Cauchy Riemann equation are satisfied at $z = 0$.	07
	b	(i)	Discuss the convergence of the series $\sum \frac{(2n)!(z-3i)^n}{(n!)^2}$	04
		(ii)	Discuss continuity of $f(z) = \begin{cases} \frac{\mathrm{Re}(z^2)}{\|z\|^2} & ; z \ne 0 \\ 0 & ; z = 0 \end{cases}$ at $z = 0$.	03
			OR	
	b	(i)	State De-Moivre's formula .Find all the root of $(8i)^{\frac{1}{3}}$ in the complex plane.	04
		(ii)	Evaluate $\int_c (x^2 - iy^2)\, dz$ along the parabola $y = 2x^2$ from $(1,2)$ to $(2,8)$.	03
Q.3	a		Using Residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$	07

Q.3 b (i) Evaluate $\int_c \frac{1+z^2}{1-z^2}\,dz$, where c is unit circle centred at 04

(1) $z=-1$

(2) $z=i$

(ii) Find the image in the w - plane of the circle $|z-3|=2$ in the z - plane under the inversion mapping $w=\frac{1}{z}$. 03

OR

Q.3 a (i) Show that $u(x,y)=x^2-y^2$ is harmonic in some domain and find the harmonic conjugate $v(x,y)$. 04

(ii) Find the Bilinear transformation which maps z $=1, i, -1$ into $\omega = 2, i, -2$. 03

b (i) Determine the poles of the function $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ and residue at each pole. Evaluate $\int_c f(z)\,dz$,where c is the circle $|z|=3$. 04

(ii) Evaluate $\int_{c:|z|=2} \frac{dz}{z^3(z+4)}$. 03

Q.4 a Find the Langrage's interpolation polynomial from the following data : 07

x	0	1	4	5
$f(x)$	1	3	24	39

Also find $f(2)$.

b (i) Using partial pivoting solve the system of equation by Gauss Elimination method : 04

$x+y+z=7$; $3x+3y+4z=24$; $2x+y+3z=16$

(ii) Find a root of the equation $x^3-4x-9=0$ using False-position method correct up to three decimal. 03

OR

Q.4 a (i) Using Newton's Backward interpolation formula , evaluate $f(300)$ from the given table: 04

x	50	100	150	200	250
$f(x)$	618	724	805	906	1032

(ii) Find real root of $x^3-5x+3=0$ correct up to three decimal using Newton-Raphson method. 03

b (i) Solve the system of equation by Gauss Seidel method 04

$10x+y+z=6$; $x+10y+z=6$; $x+y+10z=6$

(ii) Using Newton's Divided difference formula find $f(3)$ from the following table 03

x	-1	2	4	5
$f(x)$	-5	13	255	625

Q.5 a (i) Using the power method find the largest Eigen value for the matrix 07

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$

b (i) Use Runge-Kutta method of second order to find the appropriate value of $y(0.2)$ given that $\frac{dy}{dx} = x - y^2$; $y(0) = 1$ and $h = 0.1$ 04

(ii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule taking $h = \frac{1}{5}$ 03

OR

Q.5 a (i) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition $y = 1$ at $x = 0$; find y for $x = 1.0$ and h=0.25 by Euler's method 04

(ii) Using Gauss Forward interpolation formula , evaluate $f(55)$ form the given table : 03

x	40	50	60	70
$f(x)$	836	682	436	272

b (i) Evaluate $\int_0^1 \frac{dt}{1+t}$ by Gaussian formula for $n = 2$ and $n = 3$. 04

(ii) Prove that $E = e^{hD}$. 03

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