

Seat No.: _____

Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER-1st / 2nd (SPFU) EXAMINATION- WINTER 2015

Subject Code: MTH001

Date: 21/12/2015

Subject Name: Calculus

Time: 10:30am to 01:30pm

Total Marks: 70

Instructions:

1. Question No. 1 is compulsory. Attempt any four out of remaining six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1

MARKS

(a) Select the correct answer: 07

1. For which values of x , $f : R \rightarrow R$ $f(x) = |x|$ is discontinuous?
(a) $x=1$ (b) $x=-1$ (c) $x=0$ (d) $x \in R$
2. What is the center of the circle $x^2 + y^2 - 2x = 0$?
(a) $(0,1)$ (b) $(0,0)$ (c) $(1,1)$ (d) $(1,0)$
3. If $u = y^x$, then $\frac{\partial u}{\partial x}$ is
(a) xy^{x-1} (b) $y^x \log_e y$ (c) 0 (d) $y^x(1+\log_e x)$
4. Find the slope of the line tangent to the curve $y = \sin^3 x$ at the point $x = \frac{\pi}{2}$
(a) 0 (b) 1 (c) 2 (d) $\frac{\pi}{2}$
5. What is the critical point of the curve $f(x) = x^2 - 2x - 3$?
(a) 0 (b) 2 (c) 1 (d) -1
6. What is the area bounded by the curve $f(x) = \sin x$ and the lines $x = 0$ and $x = \pi$?
(a) 0 (b) 1 (c) -2 (d) 2
7. If $u = ax^2 + 2hxy + by^2$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to _____.
(a) $2u$ (b) u (c) 0 (d) $3u$

(b) Select the correct answer: 07

1. If $u = x^3 e^{-x/y}$ then $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy}$ is equal to _____.
(a) $3u$ (b) $6u$ (c) $9u$ (d) $-u$
2. If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to _____.
_____.

(a) $\frac{f_x}{f_y}$

(b) $\frac{f_y}{f_x}$

(c) $-\frac{f_y}{f_x}$

(d) $-\frac{f_x}{f_y}$

3. For which values of p , $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent?

- (a) $p < 1$ (b) $p \leq 1$ (c) $p > 1$ (d) $p \geq 1$

4. What is the degree of the homogeneous function $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$?

- (a) 0 (b) 1 (c) -1 (d) 2

5. Which series is convergent?

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$

(b) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(d) $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$

6. What is the value of $\lim_{(x,y) \rightarrow (3,-4)} \sqrt{x^2 + y^2}$?

- (a) 5 (b) 4 (c) 3 (d) 0

7. The infinite series $1 + r + r^2 + r^3 + \dots$ is convergent if

- (a) $|r| > 1$ (b) $|r| < 1$ (c) $r = 1$ (d) $r < -1$

Q.2 (a) Test the convergence of the series

03

$$\sum_{n=1}^{\infty} \frac{n^3}{4^n}$$

(b) Test the convergence of

04

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$$

Find the radius of convergence and interval of convergence of the series

07

(c) $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

Q.3 (a) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in power of $x - 3$.

03

(b) Discuss the convergence of the series

04

$$\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{5 + n^5}$$

(c) Evaluate the following integrals

07

(i) $\int_0^2 \int_0^2 (x^2 + y^2) dx dy$ (ii) $\int_0^{\frac{\pi}{2}} \int_0^{a(1+\cos\theta)} r dr d\theta$

Q.4 (a) Determine whether

03

$\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + 3y^2}$ exist or not? If it exists , find the value of the limit.

(b) If $u = r^m$, where $r^2 = x^2 + y^2 + z^2$ prove that $u_{xx} + u_{yy} + u_{zz} = m(m+1)r^{m-2}$. **04**

(c) Reverse the order of integration and evaluate the integral **07**

$$\int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y} dy dx .$$

Q.5 (a) Express $\frac{du}{dt}$ when $u = x^2 + y^2 + z^2$, $x = e^t$, $y = e^t \sin t$ and $z = e^t \cos t$. **03**

(b) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - y^2}} y^2 \sqrt{x^2 + y^2} dy dx$ by changing into polar coordinates. **04**

(c) If $u = \tan^{-1}(x^2 + 2y^2)$ then show that **07**

$$(i) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u \quad (ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u .$$

Q.6 (a) Evaluate $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} z dz dy dx$. **03**

(b) Find the equations for tangent plane and normal line at the point (1, 1, 1) **04**
on the surface $x^2 + y^2 + z^2 = 3$.

(c) Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **07**

Q.7 (a) Expand $e^x \cos y$ in powers of x and y up to second degree. **03**

(b) If $u = f(x^2 + 2yz, y^2 + 2zx)$, prove that **04**

$$(y^2 - zx) \frac{\partial u}{\partial x} + (x^2 - yz) \frac{\partial u}{\partial y} + (z^2 - xy) \frac{\partial u}{\partial z} = 0 .$$

(c) Evaluate $\iiint_v z dv$, where v is the upper half of the sphere $x^2 + y^2 + z^2 = 1$. **07**

