Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER– II• EXAMINATION – WINTER 2015

Subject Code: 4420601 **Date: 30/11/ 2015**

Subject Name: Discrete Mathematics for Computer Science

Time:02.30 pm to 05.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

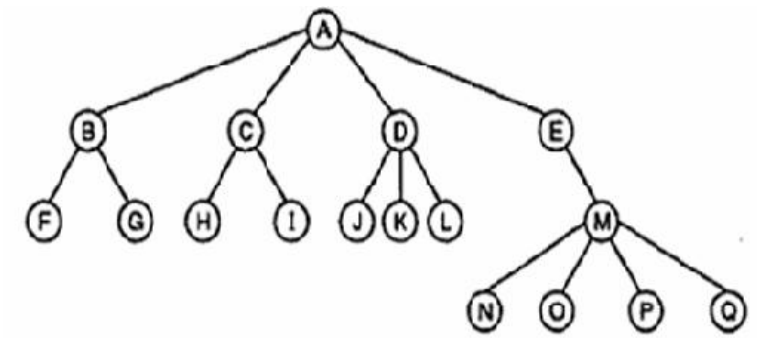
Q.1 **(a)** Define: **07**

1) Chain.
2) Lattice.
3) Partially Ordered Set
4) Tree.
5) Anti-Atoms.
6) Binary Relation
7) Isolated Node

(b) Define Sub-Lattice. Draw Hasse-Diagram and also find Cover of each of the following: **07**

1) (S_{70}, D)
2) (S_{105}, D)
3) $(P(A), \subseteq)$ Where A = {a,b,c}

Q.2 **(a)** Define Binary tree. Obtain Binary tree representation for the following: **07**



(b) Define: Directed tree, Null Graph, Forest, Elementary Path. Show by means of an example that a simple digraph in which exactly one node has in-degree 0 and every other node has in-degree 1 is not necessarily a directed tree. **07**

OR

(b) Use the Quine-McCluskey algorithm to find the prime implicants of the expression: f (a, b, c, d) = Σ (5, 7, 10, 13, 15). Also obtain a minimal expression
for the same. **07**

Q.3 **(a)** Show that the lattice $<S_n, D>$ for n = 100 is isomorphic to the direct product of lattices for n = 4 and n = 25. **07**

(b) Use the K-map representation to find a minimal sum-of-products expression for the following function: **07**

i) f (a,b,c) = Σ (0,1,4,6)

ii) f (a,b,c,d) = Σ (0,1,2,3,13,15)

OR

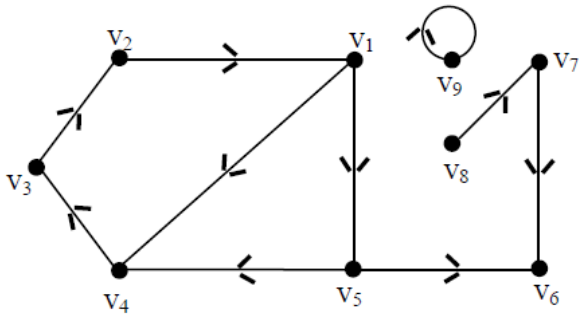
Q.3 **(a)** i. Define complemented lattice. Which of two lattices <Sn, D> for n=30 and n=45 are complemented? Draw Hasse Diagram of these lattices. Are these lattices distributive? Justify your answer **05**

ii. Consider the set B = {0, 1}. Show that <B,*, ⊕,’,0,1> is a Boolean algebra**.** **02**

(b) i. State and prove Stone’s representation theorem by giving a suitable example. **04**

ii. Explain Cosets and Lagrange’s theorem in brief. **03**

Q.4 **(a)** Define node base of a digraph. Find all node base of the digraph in the figure given below. List out some of the properties of a node base. **07**



(b) Define: **07**

i) Join irreducible elements.

ii) Atoms of a Boolean algebra.

Determine Join-irreducible elements and atoms of following Boolean algebra also draw the Hasse Diagram:

i) (S_{90}, D)

ii) ⟨ P(S), ∩,∪, ', Φ, S⟩ where S = {a, b, c}

OR

Q.4 **(a)** Define Boolean Algebra. Show that **07**

(i) $a + 0 = a$

(ii) $a + 1 = a'$

(iii) $a + a = 0$

(iv) $a + a' = 1$

where $a + b = (a * b') \oplus (a' * b)$

(b) Define Lower bound and Upper bound. Let P = <2, 3, 5, 6, 9, 15, 24, 45}, D> be a poset. Draw the Hasse diagram. Find **07**

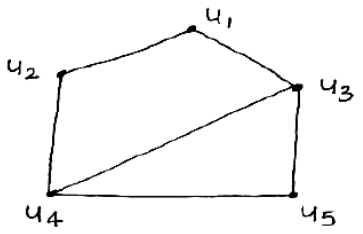
i) maximal element. & minimal element.

ii) the greatest and least element.

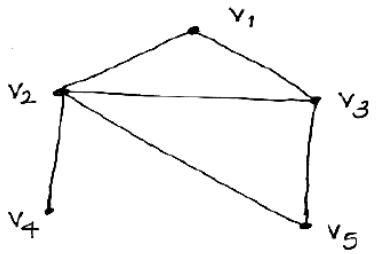
iii) the lower bounds of {9, 5}, if any & the upper bound of { 9, 15 } , if any

iv) GLB of {15, 24} & LUB of {15, 24}.

Q.5 **(a)** Define: Isomorphic Graph. Verify that, are the following graphs are isomorphic? **07**

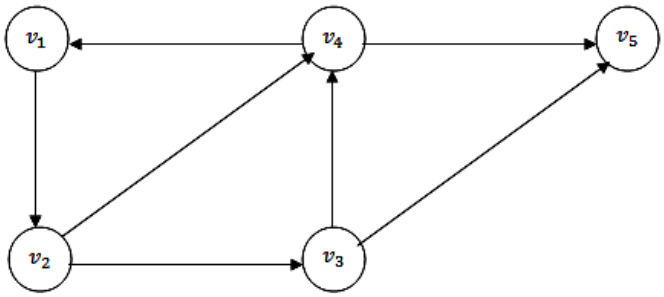


Graph – 1



Graph - 2

(b) Find the strong components of the following diagraph. Also find its unilateral and weak components. Also find weather this graph is strongly connected, unilaterally connected, weakly connected. **07**



OR

Q.5 **(a)** Define cyclic group. Show that cyclic group is abelian but converse is not true. Is <z5, +5> a cyclic group? If so, find its generators. **07**

(b) Define Group. Prove that <Z4, +4> is isomorphic to < Z5, *5> where Z5* = Z5-[0]. **07**

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