Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER II (OLD) – • EXAMINATION – SUMMER 2016

Subject Code: 1722309 **Date:21/05/2016**

Subject Name: Numerical Methods

Time:10:30 am to 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Explain the following: **07**

1) Absolute error 2) Relative Approximate Error 3) Accuracy 4) Precision 5) Round-off Error 6) Truncation Error 7) Error Propagation

(b) Discuss the steps involved in an Engineering Problem Solving. **07**

What is mathematical modeling?

Q.2 **(a)** Find a real root of the equation $e^{-x}(3.2\,sin\,x - 0.5\,cos\,x) = 0$ using Bisection method. Take [3, 4] as initial interval and go up to 5 iterations.Also, calculate percentage relative approximate error. **07**

(b) You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit. The equation that gives the minimum number of computers n to be sold after considering the total costs and the total sales is $f(n) = 40n^{1.5} - 875n + 35000 = 0$. **07**

Use the Newton-Raphson method to find the minimum number of computers that need to be sold to make a profit. (Perform three iterations). Find the percentage relative approximate error in each iteration. Take initial guess $n_0 = 50$

OR

(b) The concentration of pollutant bacteria c in a lake decreases according to $c = 75e^{-1.5t} + 20e^{-0.075t}$ .Determine the time required for the bacteria concentration to be reduced to 15 using the Newton- Raphson method with an initial guess of t=6 and a stopping criterion of 1%. **07**

Q.3 **(a)** The upward velocity of a rocket is given at three different times in Table below. **07**

Time, t(s)	Velocity, v(m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as $v(t) = a_1t^2 + a_2t + a_3, \quad 5 \le t \le 12$

The coefficients a_1, a_2 and a_3 for the above expression are given by

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Use Gauss Elimination method to find the values of a_1, a_2 , a_3.

(b) Check whether the following system is diagonally dominant or not. If not, rearrange the equations so that it becomes diagonally dominant and hence solve the system by Gauss Seidel method taking initial guesses as $x = y = z = 0$. (Perform three iterations) **07**

$$\begin{aligned} -100y + 130z &= 230 \\ -40x + 150y - 100z &= 0 \\ 60x - 40y &= 200 \end{aligned}$$

OR

Q.3 **(a)** **07**

$$\begin{aligned} 2x_1 + 100{,}000x_2 &= 100{,}000 \\ x_1 + x_2 &= 2 \end{aligned}$$

i) Solve the given set of equations using Gauss elimination and a pivoting strategy.
ii) Repeat the solution after scaling the equations.
iii) Comment on the solution which you have obtained in (i) and (ii).

(b) A river consists of a series of four reservoirs. Mass balance can be written for each reservoir and the following set of simultaneous linear algebraic equations results: **07**

$$\begin{bmatrix} 13.422 & 0 & 0 & 0 \\ -13.422 & 12.252 & 0 & 0 \\ 0 & -12.252 & 12.377 & 0 \\ 0 & 0 & -12.377 & 11.797 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 750.5 \\ 300 \\ 102 \\ 30 \end{bmatrix}$$

Where the right-hand side vector consists of the loading of chloride to each of the four lakes and c_1, c_2, c_3, c_4 = the resulting chloride concentrations for Lakes 1, 2, 3 and 4 respectively. Use Matrix Inversion method to solve for the concentrations in each of the four lakes.

Q.4 **(a)** The following table gives the age of tablet machine of certain make and annual maintenance costs. Obtain the linear regression model for cost related to age. **07**

Age of machine(in years) X:	2	4	6	8
Maintenance cost(in thousand Rs.)Y:	10	20	25	30

(b) Develop cubic splines for the data given in the following table and predict f(3.4). **07**

X:	2.5	3	4
f(X):	7	8	2

OR

Q.4 **(a)** Following values are taken from the steam table for super-heated H_2O at 200 MP_a to find the corresponding entropy s for a specific volume v of 0.108 m^3/kg using Lagrange's Interpolation formula. **07**

v[m^3/kg]	0.10377	0.11144	0.1254
s[kj/kg.K]	6.4147	6.5453	6.7664

(b) Given the data below, find the isothermal work done on the gas as it is compressed from 23L to 3L (remember that $W = -\int_{v_1}^{v_2} PdV$) **07**

V, L	3	8	13	18	23
P,atm	12.5	3.5	1.8	1.4	1.2

Find the work performed on the gas numerically using appropriate Newton-Cotes Integration formula.

Q.5 (a) Given that $\frac{dy}{dx} = x + y^2, y(0) = 1$. Use Runge-Kutta method to find approximate value of $y(0.2)$, take step-size of 0.1. **07**

(b) A steady-state heat balance for a rod can be represented as $\frac{d^2T}{dx^2} - 0.15T = 0$. Use the finite-difference approach for a 10-m rod with T(0)=240 and T(10)=150 taking $\Delta x = 2$. **07**

OR

Q.5 (a) i) What is the importance of numerical method? **07**
ii) Write an algorithm for modified Euler's method.

(b) Use Euler's method to numerically integrate the equation $\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$ from x=0 to x=2 with a step size of 0.5, the initial condition at x=0 is y=1. Recall that the exact solution is given by $y = -0.5x^4 + 4x^3 - 10x^2 + 8.5x + 1$. Calculate percentage relative true error in each step. **07**

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