Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER I (NEW) – • EXAMINATION – SUMMER 2016

Subject Code: 2710002 **Date:16/05/2016**

Subject Name: Computational Methods

Time:02:30 pm to 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**

Q.1 **(a)** Psychological tests of intelligence and of engineering ability were applied to 7 students. Here is a record of ungrouped data showing intelligent ratio (I.R.) and engineering ratio (E.R.). Calculate the coefficient of correlation. **05**

Student	A	B	C	D	E	F	G
I.R.	105	102	101	100	99	98	95
E.R.	101	100	98	95	96	104	92

(b) In sampling, a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2.Out of 1000 such samples, how many would be expected to contain at least 3 defective parts as per binomial distribution. **05**

(c) Solve exact differential equation : **04**

$$\left[\cos x \tan y + \cos(x+y)\right]dx + \left[\sin x \sec^2 y + \cos(x+y)\right]dy = 0 \cdot$$

Q.2 **(a)** If $\mathbf{F} = (2x^2 - 4z)\boldsymbol{i} - 2xy\,\boldsymbol{j} - 8x^2\boldsymbol{k}$, then evaluate $\iiint_V div\,F\,dV$, where V is bounded by the planes $x=0, y=0, z=0, x+y+z=1$. **07**

(b) A body execute damped forced vibrations given by the equation, $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 64x = 128\cos 8t$. Solve the equation, when $x = \frac{1}{3}, \frac{dx}{dt} = 0 \;\; at \;\; t = 0.$ **07**

OR

(b) Solve the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$ using Cauchy –Euler equation. **07**

Q.3 **(a)** Using convolution theorem, determine $L^{-1}\left\{\frac{1}{\left(S^2+\omega^2\right)^2}\right\}$. **07**

(b) Obtain the Fourier expansion of $\sinh ax$ in $-\pi < x < \pi$. **07**

OR

Q.3 **(a)** Using Laplace transformation obtain the solution of the $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = f(t); t \geq 0,$ where $\begin{matrix} f(t) = 3; 0 \leq t < 6 \\ f(t) = 0; t \geq 6 \end{matrix}$. **07**

(b) Solve the one-dimensional wave equation by Fourier transforms $\frac{\partial u}{dx} = \frac{1}{c^2}\frac{\partial^2 u}{dx^2}$, $-\infty < x < \infty, t > 0$ with initial conditions $u(x,0) = f(x), \frac{\partial u(x,0)}{\partial t} = g(x)$ and boundary conditions, $u, \frac{\partial u}{\partial x} \to 0 \;\; as \;\; x \to \pm\infty \cdot$ **07**

Q.4 **(a)** A team of 3 parachutists is connected by a weightless cord while free falling a velocity of 5 m/sec. Derive the expression which correlate the tension in each section of cord and acceleration of team. Solve the obtained linear equation system using Gauss Seidal Method to find tension in each section of cord and acceleration of team (accurate up to two decimal place). Start the iteration with acceleration=5 m/s^2, tension in each section of cord =30N. Do the 15 iterations. **08**

Parachutists	Mass (Kg)	Drag Coefficient (Kg/sec)
1	70	10
2	60	14
3	40	17

(b) Obtain largest eigen value of matrix A using X_1 and X_2 as starting vector. Is it same? If not why? **06**

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

OR

Q.4 **(a)** Solve the following linear equation system using Gauss Elimination Method. **07**

$$\begin{aligned} 10x - 7y + 3z + 5u &= 6 \\ -6x + 8y - z - 4u &= 5 \\ 3x + y + 4z \mp 11u &= 2 \\ 5x - 9y - 2z + 4u &= 7 \end{aligned}$$

(b) Find the Eigen values of matrix A using diagonalization. The X_1, X_2, X_3 are the Eigen vectors of matrix A. **07**

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Q.5 **(a)** A function $f(x)$ has the values, **07**

x	0.1	0.2	0.3	0.4
$f(x)$	0.76	0.58	0.44	0.35

Obtain a least square fit to above data of the form $f(x) = a\ b^x$.

(b) Solve the 2nd order differential equation $\frac{d^2T}{dx^2} + h'(T_a - T) = 0$ by finite difference method for 10 meter rod with $h' = 0.01\text{m}^{-2}$, $T_a = 20$. Boundary conditions are $T(0) = 40$, $T(10) = 200$. Take a step size = 2. **07**

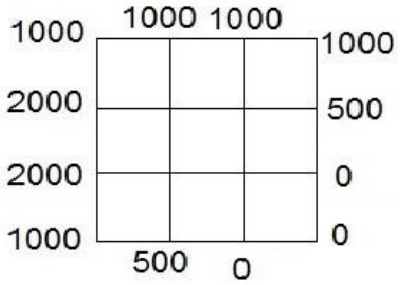
OR

Q.5 **(a)** Obtain natural cubic spline for given data set, **07**

x	2	3	4
y	11	49	123

Calculate (1) y (2.5) (2) y'(2).

(b) Given that the values of $u(x,y)$ on the boundary condition of square as shown in figure below, evaluate the function $u(x,y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of figure by Gauss Jacobi Method. **07**



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