Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER I (NEW) – • EXAMINATION – SUMMER 2016

Subject Code: 2710710 **Date:16/05/2016**

Subject Name: Applied Linear Algebra

Time:02:30 pm to 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** **(1)** Express the vector $v = (6,11,6)$ as a linear combination of $v_1 = (2,1,4)$, $v_2 = (1,-1,3)$ and $v_3 = (3,2,5)$ **07**

(2) Determine whether the following set of vectors form a basis for R^3.
$(1,1,1),\ (1,\ 2,\ 3),\ (2,-1,1)$

(b) Check whether the following are subspace of R^3. Justify your answer. State all possible subspaces of R^3. **07**

(1) $W = \{(x,0,0 \mid x \in R\}$
(2) $W = \{(x,y,z \mid x^2 + y^2 + z^2 \leq 1\}$
(3) $W = \{(x,y,z \mid y = x + z + 1\}$

Q.2 **(a)** Determine whether the following functions are linear transformation : **07**

(1) $T: M_{mn} \to M_{nm}$, where $T(A) = A^T$
(2) $T: M_{22} \to R, T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a^2 + b^2$

(b) Let $T: R^3 \to R^3$ be multiplication by A. Determine whether T has an inverse. If so, find $T^{-1}\left(\left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right]\right)$ where $A = \begin{bmatrix} 1 & 4 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ **07**

OR

(b) Let $T_1: R^2 \to R^2$ and $T_2: R^2 \to R^2$ be the linear operators given by the formula $T_1(x,y) = (x+y, x-y)$ and $T_2(x,y) = (2x+y, x-2y)$ **07**
(i) Show that T_1 and T_2 are one to one.
(ii) Find the formula for $T_1^{-1}(x,y)$ and $T_2^{-1}(x,y)$ and $(T_2 \circ T_2)^{-1}(x,y)$

Q.3 **(a)** If $T: R^2 \to R^3$ be the linear transformation defined by **07**

$$T\left(\left[\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$$

Find the matrix of the transformation T with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3, where

$$u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(b) Find the eigen values and eigen vectors the following matrix A. **07**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

OR

Q.3 **(a)** Let $T: R^4 \to R^3$ be the linear transfomration given by the formula $T(x_1, x_2, x_3, x_4)$=$(4x_1 + x_2 - 2x_3 - 3x_4, 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4)$ **07**

(1) Find a basis for $\ker(T)$.

(2) Find a basis for $R(T)$.

(b) Verify cayley-Hamiton theorem for the following matrix and hence, find A^{-1}. **07**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Q.4 **(a)** Find an orthogonal matrix P that diagonalizes **07**

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

(b) The vector space R^3 with the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectros $u_1 = (1,1,1), u_2 = (0,1,1), \quad u_3 = (0,0,1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$. **07**

OR

Q.4 **(a)** Define diagonalize of matrix A. Prove that if A is an $n \times n$ matrix, then the following are quivalent. **07**

(1) A is diagonalizable.

(2) A has a linearly independent eigenvectors.

(b) Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix **07**

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}.$$ Find the minimal polynomial for T

Q.5 **(a)** Define the following terms : **07**

1. Orthogonal matrix
2. Proper and improper orthogonal matrix
3. Unitary matrix
4. Irreducible polynomial
5. Annihilating polynomial
6. Minimum polynomial

(b) Find the Jordan forms of **07**

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

OR

Q.5 **(a)** Solve the following system by Gauss-Jordan elimination method: **07**

$x_1 + x_2 + 2x_3 = 8, -x_1 - 2x_2 + 3x_3 = 1, 3x_1 - 7x_2 + 4x_3 = 10$

(b) Find a basis for the row and column spaces of **07**

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

****