Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER I (NEW) – • EXAMINATION – SUMMER 2016

Subject Code: 2712704 **Date:18/05/2016**

Subject Name: First Course in Optimization Techniques

Time:02:30 pm to 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find the maxima or minima, if any, of the function **04**

$$f(x) = -(x-1)^3(x+1)^2.$$

(ii) Find the critical points of the function

$$f(x,y) = x^2y^2 - 5x^2 - 8xy - 5y^2.$$ **03**

(b) Two towns are to get their water supply from a river. Both towns are on the same side of the river at a distance of 6 km and 18 km from the river bank. If the distance between the points on the river bank nearest to the towns respectively be 10 km, find **07**

(i) where a single pumping station may be located to require the least amount of pipe,

(ii) how much pipe is needed.

Q.2 **(a)** Define a convex function. How will you check that a function is convex or not? Check whether the function **07**

$$f(x_1,x_2) = 4x_1^2 + 3x_2^2 + 5x_3^2 + 6x_1x_2 + x_1x_3 - 3x_1 - 2x_2 + 15$$

is convex.

(b) Minimize $f = -x_1 + x_2 - 3x_3$ **07**

subject to

$$x_1 + x_2 + x_3 \le 10,$$
$$2x_1 - x_3 \le 2,$$
$$2x_1 - 2x_2 + 3x_3 \le 0,$$
$$x_i \ge 0, \quad i = 1,2,3$$

by Simplex method.

OR

(b) Find the solution of the following LP problem graphically: **07**

Minimize $f = -x_1 + 2x_2$

subject to

$$-x_1 + 3x_2 \le 10,$$
$$x_1 + x_2 \le 6,$$
$$x_1 - x_2 \le 2,$$
$$x_1 \ge 0, \qquad x_2 \ge 0.$$

Q.3 **(a)** Use Dichotomous Search Method to find the minimum of **07**

$$f(\lambda) = \frac{\lambda}{2} + \frac{2}{\lambda}$$

in the interval $(0.00, 3.00)$ to within 10% of the exact value. Take $\delta = 0.001$.

(b) Use Quasi-Newton Method to minimize the function **07**

$$f(\lambda) = \lambda\sqrt{4 - \lambda^2}.$$

Take $\lambda_1 = 1.6$, the step size $\Delta\lambda = 0.01$ and $\varepsilon = 0.01$. Perform only two iterations.

OR

Q.3 **(a)** Use Fibonacci Method with $n = 6$ to minimize the function **07**

$$f(\lambda) = 0.65 - \frac{0.75}{1+\lambda^2} - 0.65\,\lambda\, tan^{-1}\frac{1}{\lambda}$$

in the interval $[0, 3]$.

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with the starting point $X_1 = [0\ 0]^T$ using Powell's method. **07**

Q.4 **(a)** Explain the terms: Design vector, side constraint, behavior constraint, Integer Programming problem **07**

(b) Define gradient of a function. How gradient is useful in optimization? Write Cauchy's steepest descent algorithm. **07**

OR

Q.4 **(a)** Use Interval Halving Method to minimize the function **07**

$$f(\lambda) = \lambda^5 - 5\lambda^4$$

with $n = 7$ in the interval $(2, 5)$.

(b) Write the general form of an optimization problem. Write some applications of optimization problem. Explain one application in detail. **07**

Q.5 **(a)** Use Fletcher Reevesmethod to minimize **07**

$$f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$$

from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform only two iterations.

(b) How transformations are useful in constrained optimization? List some transformations normally used in optimization. **07**

OR

Q.5 **(a)** Write the algorithm of Random Walk method. **07**

(b) Write the algorithm of sequential linear programming. **07**

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