Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER II (NEW) – • EXAMINATION – SUMMER 2016

Subject Code: 2720501 **Date: 24/05/2016**

Subject Name: Statistical Signal Analysis

Time:10:30 am to 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) A six-sided die is tossed 12 times. How many distinct sequences of faces (numbers from the set {1,2,3,4,5,6}) have each number appearing exactly twice? What is the probability of obtaining such a sequence? **04**

(ii) Define conditional probability and explain with example. **03**

(b) A manufacturing process produces a mix of "good" memory chips and "bad" memory chips. The lifetime of good chips follow the exponential law, with a rate of failure α. The lifetime of bad chips also follow the exponential law, but the rate of failure is 1000α. Suppose that the fraction of good chips is 1-*p* and of bad chips, *p*. **07**

(1) Find the probability that a randomly selected chip is still functioning after t seconds.

(2) Suppose that in order to "weed out" the bad chips, every chip is tested for t seconds prior to leaving the factory. The chips that fail are discarded and the remaining chips are sent out to customers. Find the value of t for which 99% of the chips sent out to customers are good.

Q.2 **(a)** (1) State and explain conditions of events A and B to be statistically independent. **03**

(2) A ball is selected from an urn containing two black balls, numbered 1 and 2, and two white balls numbered 3 and 4. Let the events A, B, and C is defined as follows:

A = {(1,b), (2,b)}, "black ball selected"; **04**

B= {(2,b), (4,w)}, "Even Numbered ball selected";

C= {(3,b), (4,w)}, "number of ball is greater than 2";

Are events A and B independent? Are events A and C independent?

(b) (i) A binary communications channel introduces a bit error in a transmission with probability p. Let X be the number of errors in n independent transmissions. Find the pmf of X. Find the probability of one or fewer errors. **03**

(ii) The transmission time X of messages in a communication system obeys the exponential probability law with parameter λ, that is **04**

$P[X>x]=e^{-\lambda x}$ where $x>0$. Find

(1) CDF of X

(2) $P[T<X\leq 2T]$ where $T=1/\lambda$

OR

(b) A production line manufactures 1000 Ω resistors that have 10% tolerance. Let X denotes the resistance of the resistor. Assuming X is a normal random variable with mean 1000 and variance 2500, find the probability that resistor picked at random will be selected. **07**

Q.3 **(a)** Show that the poisson distribution can be used as a convenient approximation to the binomial distribution for large *n* and small *p* **07**

(b) (i) Find E[E[Y|X]], where E[.] is expected operator **03**

(ii) Compute the conditional means for given conditional pdfs. **04**

$$f_{Y|X}(y|x) = \frac{1}{x} \qquad y \le x < 1, 0 < x < 1$$

$$f_{X|Y}(x|y) = \frac{1}{1-y} \qquad y \le x < 1, 0 < x < 1$$

OR

Q.3 **(a)** Define and explain Mean Square Derivative of random process X(t) **07**

(b) Let X be a continuous random variable with the pdf **07**

$$f_X(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x < 0 \end{cases}$$

Find the transformation Y = g(X) such that the pdf of Y is

$$f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Q.4 **(a)** Let X and Y be the bivariate random variable with joint pdf **07**

$$f_{XYZ}(x,y,z)=ce^{-x}e^{-y} \qquad 0 \le y \le x < \infty$$

Find (1) Normalized Constant (2) Marginal PDF (3) Independency

(b) Derive Power Spectral Density of output when random process x(t) passes through LTI system having impulse response *h(t)* **07**

OR

Q.4 **(a)** Define and explain Ergodic process. Explain concept of ensemble average and time average. **07**

(b) Let (*X,Y,Z*) be a trivariate random variable with joint pdf **07**

$$f_{XYZ}(x,y,z)=ke^{-(ax+by+cz)} \text{ for } x>0,\ y>0 \text{ and } z>0$$

where $a,b,c > 0$ and k is constant

(a) Determine the value of *k*

(b) Find the marginal joint pdf of *X* and *Y*

(c) Find the marginal pdf of *X*

(d) Are *X,Y* and *Z* independent?

Q.5 **(a)** Explain Joint Moments, Correlation, and Covariance. Also prove that covariance of the independent random variable is zero. **07**

(b) Explain Mean Square Continuity in context of random process **07**

OR

Q.5 **(a)** Find the mean and variance of M(t), the moving average over half a period of a random amplitude sinusoid X(t) with period T: **07**

$$M(t) = \frac{2}{T}\int_{t-T/2}^{t} X(t')\,dt'$$

(b) Show that a WSS random process *X(t)* is mean square continuous if and only if its autocorrelation function $R_X(\tau)$ is continuous at $\tau = 0$. **07**

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