Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER II (NEW) – • EXAMINATION – SUMMER 2016

Subject Code: 2720714 Date: 24/05/2016

Subject Name: Modern Control System

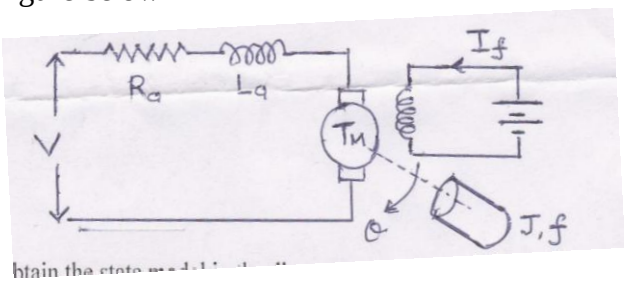
Time:10:30 am to 01:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Discuss stability in the sense of Liapunov. Explain asymptotic stability and asymptotic stability in the large with the help of appropriate diagram and relevant equations. **07**

(b) Obtain the state space model for the separately excited d.c.motor shown in figure below **07**



Q.2 **(a)** Obtain the state model in the diagonal form for the transfer function given by $G(s) = (s+2) / (s^3 + 9s^2 + 20s)$. Is this system controllable ? **07**

(b) Discuss the advantages and disadvantages of state space model for a system compared to transfer function model. Hence, define state and state space. **07**

OR

(b) Define 'state transition matrix'. Hence, derive the solution to the vector equation $\dot{\boldsymbol{X}} = A\boldsymbol{X} + Bu$ **07**

Q.3 **(a)** A system is described by $\dot{\boldsymbol{X}} = A\boldsymbol{X} + Bu$ where $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Obtain the solution for $\boldsymbol{X}(t)$ when the system is subject to unit step input given $\mathbf{X}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ **07**

(b) Define 'Controllability'. Discuss the Gilbert's method to test the controllability for a system given by $\dot{\boldsymbol{X}} = A\boldsymbol{X} + Bu$ **07**

OR

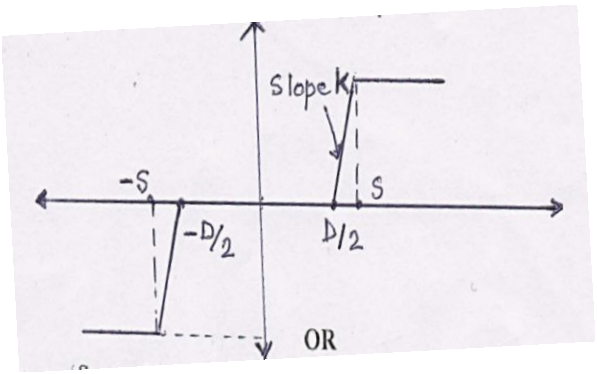
Q.3 **(a)** Define Positive Definiteness, Positive Semi Definiteness and Indefiniteness of a quadratic function. Hence, check the definiteness of the following function **07**

1. $Q_1 = (3x_1 - 2x_2)^2$
2. $Q_2 = 2(x_1 - \frac{1}{2}x_2)^2 + \frac{3}{2}(x_2 - \frac{2}{3}x_3)^2 + \frac{4}{3}x_3^2$

(b) Derive the Riccati equation for Linear Quadratic Regulator problem. **07**

Q.4 **(a)** Explain how the Phase plane method is useful in determining the behavior of nonlinear systems. **07**

(b) Obtain the describing function for the following nonlinearity (dead zone with saturation). Obtain its response to sinusoidal input. **07**



OR

Q.4 **(a)** What is a 'State Observer'. Obtain the state space representation for a controller-observer based system & show that the pole placement design & observer design are independent of each other. **07**

(b) Obtain the observer design for the system given below. It is desired that the observer eigenvalues are placed at (-50, -50) **07**

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & -4\\ 1 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2\\ 0\end{bmatrix} u \text{ and } y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Q.5 **(a)** Determine the smooth curve with the smallest length connecting point.x (0) =1 to the line t =5 **07**

(b) Derive the Euler Langrange equation for a fixed end point problem. **07**

OR

Q.5 **(a)** Consider an autonomous system represented by $\dot{x}$ = **F**(*x*). For a second order system, explain the nature of the phase plane trajectories when i. eigenvalues are real, distinct & negative ii. eigenvalues are complex conjugates with negative real parts iii. eigenvalues are imaginary & hence discuss the stability of its singular point. **07**

(b) Consider a inverted pendulum whose system dynamics are defined as **07**

$\ddot{\theta} = \omega_n^2\theta - u, \omega_n^2 = g/L$.

The system linearised about the vertical axis is given by

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ \omega_n^2 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ -1\end{pmatrix}u,$$ Find the control signal that stabilizes the system that minimizes the performance index

$$J = \frac{1}{2}\int_0^\infty \left(\theta^2 + \frac{1}{c^2}u^2\right)dt, \quad Q = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \quad R = \frac{1}{c^2}$$

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