Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER II (NEW) – • EXAMINATION – SUMMER 2016

Subject Code: 2724712 **Date: 02/06/2016**

Subject Name: Optimization Theory and Practice

Time:10:30 am to 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Two types of printed circuit boards A and B are produced in a computer manufacturing company. The component placement time, soldering time and inspection time required in producing each unit of A and B are given below: **07**

Circuit Board	Time Required per Unit (min) for: Component Placement	Soldering	Inspection
A	16	10	4
B	10	12	8

If the amounts of time available per day for component placement, soldering and inspection are 1500, 1000 and 500 person-minutes, respectively, determine the number of units of A and B to be produced for maximizing the production by Simplex Method.

(b) Write the Simulated Annealing algorithm. Also draw its flow chart. How it differs from Genetic Algorithm? **07**

Q.2 **(a)** Use Powell's method to minimize the function **07**

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

starting from the point $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Take $\varepsilon = 0.01$.

(b) Minimize $F = 2x_1 + 2x_2$ **07**

subject to

$$x_1 + 2x_2 \geq 1, \qquad 2x_1 + x_2 \geq 1$$
$$x_1 \geq 0, \qquad x_2 \geq 0$$

by Dual Simplex Method.

OR

(b) Use Graphical Method to solve the following problem: **07**

Minimize $f = x + 3y$

subject to

$$-4x + 3y \leq 12, \qquad x + y \leq 7, \qquad x - 4y \leq 2,$$

x and y are unrestricted in sign.

Q.3 **(a)** (i) Find the points at which the function **03**

$$y = x^3 + 6x^2 - 15x + 5$$

has maximum and minimum values.

(ii) Write some engineering applications of optimization. Describe one of them in detail. **04**

(b) (i) Consider the following optimization problem: **03**

$$Minimize\ f(X) = (x_1 - 3)^2 + (x_2 - 2)^2$$

subject to

$$5 \geq x_1 + x_2, \qquad x_1^2 - 2x_2 \geq 0.$$

Use Kuhn-Tucker conditions to check whether $\boldsymbol{X} = [2, 3]^T$ is a local minimum.

(ii) Find the extreme points of the function **04**

$$f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6.$$

OR

Q.3 **(a)** (i) In a submarine telegraph cable, the speed of signaling varies as $x^2 log(1/x)$, where x if the ratio of the core to that of the covering. Find x which maximizes the speed. **03**

(ii) Explain the terms: side constraint, behavioral constraint, geometric programming problem, constraint surface **04**

(b) (i) Determine whether the following matrix is positive definite, negative definite or indefinite by evaluating the signs of its submatrices: **03**

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}.$$

(ii) Use Lagrange's multiplier method to find the critical points of the function $f(x, y, z) = x^2 + y^2 + z^2$ given that $z^2 = xy + 1$. **04**

Q.4 **(a)** Find the minimum of $f(x) = x(x - 1.5)$ in the interval $(0.0, 1.00)$ to within 10 % of the exact value using Dichotomous Search Method. **07**

(b) Find the minimum of the function **07**

$$f(\lambda) = 0.65 - \frac{0.75}{1 + \lambda^2} - 0.65\lambda tan^{-1}(1/\lambda)$$

using the secant method with an initial step size of $t_0 = 0.1, \lambda_1 = 0.0$ and $\varepsilon = 0.01$.

OR

Q.4 **(a)** Minimize the function $f(x) = x^3 - 3x$ in the interval $(0.0, 2.0)$ using the Fibonacci Method with $n = 6$. **07**

(b) Find the minimum of the function $f(\lambda) = \lambda^5 - 5\lambda^3 - 20\lambda + 5$ using Quasi-Newton Method with the starting point $\lambda_1 = 2.2$ and the step size $\Delta\lambda = 0.01$. Perform three iterations only. **07**

Q.5 **(a)** Write the algorithm of Univariate method. Perform only one iteration of Univariate Method to minimize **07**

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

with the starting point $\boldsymbol{X}_1 = [0, 0]^T$. Take $\varepsilon = 0.01$.

(b) In case of Interior Penalty Function Method, answer the following questions:

(i) How will you select the initial value of the Penalty Parameter? **02**

(ii) What is normalization of constraints? What is its importance? **02**

(iii) What are the different ways to check the convergence? **03**

OR

Q.5 **(a)** (i) Transform the following constrained problem into an equivalent unconstrained problem: **03**

$$Minimize\ f(x_1, x_2) = x_1^2 - x_2^2$$

subject to $-1 \le x_1 \le 1,\ 2 \le x_2 \le 10$.

(ii) Use Newton's method to minimize the function **04**

$$f(x_1, x_2) = 3x_1^2 - 2x_1x_2 + x_2^2 - 2x_1 - 3x_2$$

by taking the starting point as $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

(b) Linearize the constraint $3x_1^2 - 2x_1x_2 + 3x_2^2 - 1 \le 0$ at the point $\boldsymbol{X}_1 = [-1\ \ 1]^T$. Also write Sequential Linear Programming Method algorithm for constrained optimization. **07**

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