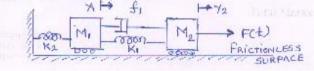
# **GUJARAT TECHNOLOGICAL UNIVERSITY** ME – SEMESTER I (OLD) – • EXAMINATION – SUMMER 2016

Date:18/05/2016 Subject Code: 710703N Subject Name: Modern Control Systems Time:02:30 pm to 05:00 pm **Total Marks: 70 Instructions:** 1. Attempt all questions. Make suitable assumptions wherever necessary. 2. 3. Figures to the right indicate full marks. Discuss the advantages and disadvantages of state space model for a system **Q.1** 07 (a) compared to transfer function model. Hence, define state and state space. (b) Explain eigenvalues and eigenvectors of a matrix. Determine eigenvalues of matrix  $A = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ 21 04 + 033 Prove that eigenvalues are invariant under a linear transformation **O.2** (a) Obtain the state model in the diagonal form for the transfer function given by 07  $G(s) = (s+2) / (s^3 + 9s^2 + 20s)$ . Is this system controllable ?

(b) Obtain the state space model for the mechanical system shown in figure. 07



### OR

- (b) Obtain the co-relation between the state space equation and transfer function. Hence, obtain the transfer function for the system given in state model form as  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u \text{ and } y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
- Q.3 (a) Define 'state transition matrix '. Hence derive the solution to the vector equation 07  $\dot{X} = AX + Bu$ 
  - (b) Check for the controllability and the observability of the system given by

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$
$$[y_1] = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

## OR

Q.3 (a) Obtain the time response of the system given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \text{ where u is unit step input and } x^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

- (b) Explain Cayley Hamilton Theorem and discuss how it can be used to find the **07** state transition matrix.
- Q.4 (a) What is a state controller? With the help of block diagram explain full order

07

07

state controller.

(b) Explain in brief the concepts and definition of Controllability and Observability. 07

#### OR

Q.4 (a) Design a state feedback controller gain using pole placement technique for the 07 system given by  $[x^2, 1] = [0]$ 

 $\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}$ 

The desired pole location of the closed loop system are s = -3 and s = -5. Use both Ackerman's formula and direct substitution method to determine the controller gain matrix **K**.

- (b) Discuss necessary and sufficient condition for state observation and hence 07 explain the design procedure of a full state observer.
- Q.5 (a) Discuss stability in the sense of Liapunov. Explain asymptotic stability and asymptotic stability in the large with the help of appropriate diagram and relevant equations.
  - (b) Define Positive Definiteness, Positive Semi Definiteness and Indefiniteness of a quadratic function. Hence, check the definiteness of the following function
    - 1.  $Q_1 = (3x_1 2x_2)^2$ 2.  $Q_2 = 2(x_1 - \frac{1}{2}x_2)^2 + \frac{3}{2}(x_2 - \frac{2}{3}x_3)^2 + \frac{4}{3}x_3^2$

#### OR

Q.5 (a) Apply Krasovski method to assess stability of the equilibrium point x(0) of the o7 system given below

$$\dot{x}_1 = -x_1$$

 $\dot{x}_2 = x_1 - x_2 - (x_2)^3/3$ 

(b) Explain the direct method of Liapunov to determine the stability of linear 07 systems. Hence, determine the stability of the system given by

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

\*\*\*\*\*

A	× ⊨ fi	H+Y2 Detai Marka
and los	M, The	M2 F(t)
K	L DOG KI	FRICTIONLESS SURPACE