Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**ME – SEMESTER I (OLD) – • EXAMINATION – SUMMER 2016**

**Subject Code: 710904N** **Date:20/05/2016**

**Subject Name: Optimization Techniques**

**Time:02:30 pm to 05:00 pm** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Explain classification of optimization problems based on **07**

(i) The existence of constraints
(ii) The nature of the design variables
(iii) The physical structure of the problem

**(b)** Explain single variable optimization and multivariable optimization with no constraints. **07**

**Q.2** **(a)** Four identical helical springs are used to support a milling machine weighing 5000 lb. Formulate the problem of finding the wire diameter (d), coil diameter (D) and the number of turns (N) of each spring (Fig. 1) for minimum weight by limiting the deflection to 0.1 in. and the shear stress to 10,000 psi in the spring. In addition, the natural frequency of vibration of the spring is to be greater than 100 Hz. The stiffness of the spring (k), the shear stress in the spring (τ), and the natural frequency of vibration of the spring ($f_n$) are given by **07**

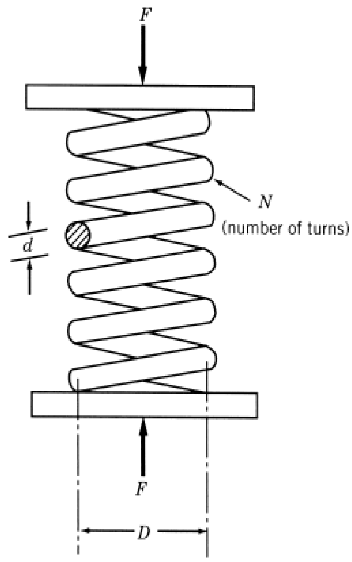

Fig. 1. Helical spring.

$$k = \frac{d^4 G}{8D^3 N}$$

$$\tau = k_s \frac{8FD}{\pi d^3}$$

$$f_n = \frac{1}{2}\sqrt{\frac{kg}{w}} = \frac{1}{2}\sqrt{\frac{d^4 G}{8D^3 N}\frac{g}{\rho(\pi \mathrm{d}^2/4)\pi DN}} = \frac{\sqrt{Ggd}}{2\sqrt{2\rho}\pi D^2 N}$$

Where G is the shear modulus, F the compressive load on the spring, w the weight of the spring, ρ the weight density of the spring, and $k_s$ the shear stress correction factor. Assume that the material is spring steel with G = 12 x $10^6$ psi and ρ = 0.3 lb/in$^3$, and the shear stress correction factor is $k_s \approx 1.05$.

**(b)** State the necessary and sufficient conditions for the maximization of a multivariable function f(x). **07**

**OR**

**(b)** Determine the maximum and minimum values of the function **07**

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$$

**Q.3** **(a)** Minimize $f(x_1, x_2) = (x_1 - 1)^2 + x_2^2$ **07**

Subject to $g_1(x_1, x_2) = x_1^3 - 2x_2 \le 0$

$g_2(x_1, x_2) = x_1^3 + 2x_2 \le 0$

Determine whether the constraint qualification and the Kuhn-Tucker conditions are satisfied at the optimum point.

**(b)** Find the extreme points of the function **07**

$f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$

**OR**

**Q.3** **(a)** Find the maximum of the function $f(X) = 2x_1 + x_2 + 10$ subject to $g(X) = x_1 + 2x_2^2 = 3$ using the Lagrange multiplier method. Also find the effect of changing the right-hand side of the constraint on the optimum value of f. **07**

**(b)** By using direct substitution method, find the dimensions of a box of largest volume that can be inscribed in a sphere of unit radius. **07**

**Q.4** **(a)** Find the dimensions of a cylindrical tin (with top and bottom) made up of sheet metal to maximize its volume such that the total surface area is equal to $A_0 = 24\pi$. **07**

**(b)** What is duality in linear programming? Describe relations between the primal and dual problems. **07**

**OR**

**Q.4** **(a)** Minimize: $f(x) = (100 - x)^2$ over (60,150) by golden section search method. **07**

**(b)** Explain dynamic programming. How is it different from linear programming? **07**

**Q.5** **(a)** Use the Simplex method to find the maximum value of objective function **07**

$z = 2x_1 - x_2 + 2x_3$

subject to the constraints

$2x_1 + x_2 \le 10$

$x_1 + 2x_2 - 2x_3 \le 20$

$x_2 + 2x_3 \le 5$

Where $x_1 \ge 0$, $x_2 \ge 0$ and $x_3 \ge 0$.

**(b)** How is the degree of difficulty defined for a constrained geometric programming problem? **07**

**OR**

**Q.5** **(a)** Solve by cutting plane method. **07**

Minimize $f(x) = -x_1 - x_2$

Subject to:

$g_1(x) = 2x_1 - x_2^2 - 1 \ge 0$

$g_2(x) = 9 - 0.8x_1^2 - 2x_2 \ge 0$

$0 \le x_1 \le 5$ and $0 \le x_2 \le 4$

**(b)** Explain branch and bound method used to solve mixed integer linear and non-linear programming methods. **07**

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