Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER I (OLD) – • EXAMINATION – SUMMER 2016

Subject Code: 714704 **Date: 20/05/2016**

Subject Name: Optimization Theory and Practice

Time:02:30 pm to 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** A manufacturing firm produces two machine parts using lathes, milling machines and grinding machines. The different machining times required for each part, the machining times available, and the profit on each machine part are given in the following table. **07**

Type of machine	Machining Time (min)		Maximum time available (min)
	Part I	Part II	
Lathes	10	5	2500
Milling Machines	4	10	2000
Grinding Machines	1	1.5	450
Profit per unit	50	100	

Use Simplex Method to determine the number of parts I and II to be manufactured to maximize the profit.

(b) Write the algorithm of Simulated Annealing. How it differs from Genetic Algorithm? **07**

Q.2 **(a)** Use Cauchy's steepest descent method to minimize the function **07**

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

starting from the point $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform only two iterations.

(b) Minimize $F = 20x_1 + 16x_2$ **07**

subject to

$$x_1 \geq 2.5$$
$$x_2 \geq 6$$
$$2x_1 + x_2 \geq 17$$
$$x_1 + x_2 \geq 12$$
$$x_1 \geq 0, \qquad x_2 \geq 0$$

by Dual Simplex Method.

OR

(b) Solve the following problem graphically: **07**

Maximize $F = x + 3y$

subject to

$$-4x + 3y \leq 12$$
$$x + y \leq 7$$
$$x - 4y \geq 2$$
$$x \geq 0, \qquad y \geq 0.$$

Q.3 **(a)** (i) Write some engineering applications of optimization. Explain one of them in detail. **05**

(ii) Find the optimum point, if any, of the function **02**

$$f(x) = x^3 - 6x^2 + 12x - 8.$$

(b) (i) Find the optimum points, if any, of the following function: **04**

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy.$$

(ii) Consider the following optimization problem: **03**

$$Minimize\ f(\boldsymbol{X}) = (x_1 - 2)^2 + (x_2 - 1)^2$$

subject to

$$2 \geq x_1 + x_2,\ x_2 \geq x_1^2.$$

Use Kuhn-Tucker condition to find whether $\boldsymbol{X} = [1\ \ 1]^T$ is a local minimum.

OR

Q.3 **(a)** (i) Give classification of optimization problems. **05**

(ii) Determine whether the following matrix is positive definite, negative definite or indefinite: **02**

$$A = \begin{bmatrix} 4 & 2 & -4 \\ 2 & 4 & -2 \\ -4 & -2 & 4 \end{bmatrix}.$$

(b) (i) An open rectangular box is to be manufactured from a given amount of sheet metal (area S). Find the dimensions of the box to maximize the volume. **04**

(ii) Find the maximum of the function $f(\boldsymbol{X}) = 2x_1 + x_2 + 5$ subject to $g(\boldsymbol{X}) = x_1 + 2x_2^2 = 3$ using the Lagrange's multiplier method. **03**

Q.4 **(a)** Use Dichotomous Method to find the minimum of $f(x) = x(x - 1.5)$ in the interval $(0.00, 1.00)$ to within 10 % of the exact value. **07**

(b) Use Newton's Method to find the minimum of the function **07**

$$f(\lambda) = \lambda^5 - 5\lambda^3 - 20\lambda + 5$$

with the starting point $\lambda_1 = 1.6$. Use $\varepsilon = 1$ for checking the convergence.

OR

Q.4 **(a)** Use Fibonacci Method to minimize **07**

$$f(x) = 0.65 - \frac{0.75}{1 + x^2} - 0.65x tan^{-1}\left(1/x\right)$$

in the interval $[0, 3]$ using $n = 6$.

(b) Find the minimum of the function **07**

$$f(\lambda) = \lambda^2 + \frac{54}{\lambda}$$

using Quasi-Newton Method with the starting point $\lambda_1 = 2.5$ and the step size $\Delta\lambda = 0.01$ in central difference formulas. Take $\varepsilon = 0.05$.

Q.5 **(a)** (i) Write the algorithm of Fletcher-Reeves Method. **03**

(ii) Write the algorithm of Univariate method. **04**

(b) Write some of the interior and exterior penalty functions generally used in constrained optimization. Also, write the algorithm of interior penalty function method. **07**

OR

Q.5 **(a)** (i) Use Newton's method to minimize the function **03**

$$f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$$

by taking the starting point as $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

(ii) Write the algorithm of Random Walk Method. **04**

(b) Linearize the constraint $6x_1^2 - x_1x_2 + 4x_2^2 - 1 \leq 0$ at the point $\boldsymbol{X}_1 = [-1\ \ 1]^T$. Also write Sequential Linear Programming Method algorithm for constrained optimization. **07**

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