Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER III (OLD) – • EXAMINATION – SUMMER 2016

Subject Code: 730403 **Date:05/05/2016**

Subject Name: Optimization Techniques

Time:10:30 am to 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find the maxima or minima, if any, of the function **04**

$$f(x) = 4x^3 - 18x^2 + 27x - 7.$$

(ii) Find the critical points of the function

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy.$$ **03**

(b) A window is in the form of a rectangle surmounted by a semi-circle. If the perimeter of the window is $100\,m$, find the dimensions of the window so that maximum light enters through the window. **07**

Q.2 **(a)** Define a convex function. How will you check that a function is convex or not? Check whether the function $f(x_1, x_2) = 2x_1^3 - 6x_2^2$ is convex. **07**

(b) Minimize $f = -x_1 - 2x_2 - x_3$ **07**

subject to

$$2x_1 + x_2 - x_3 \leq 2,$$
$$2x_1 - x_2 + 5x_3 \leq 6,$$
$$4x_1 + x_2 + x_3 \leq 6,$$
$$x_i \geq 0, \quad i = 1, 2, 3$$

by Simplex method.

OR

(b) Find the solution of the following LP problem graphically: **07**

Maximize $f = 2x_1 + 6x_2$

subject to

$$-x_1 + x_2 \leq 1,$$
$$2x_1 + x_2 \leq 2,$$
$$x_1 \geq 0, \qquad x_2 \geq 0.$$

Q.3 **(a)** Use Dichotomous Search Method to find the minimum of **07**

$$f(\lambda) = \lambda(\lambda - 1.5)$$

in the interval $(0.00, 1.00)$ to within 10% of the exact value. Take $\delta = 0.001$.

(b) Use Newton-Raphson Method to minimize the function **07**

$$f(\lambda) = \frac{\lambda}{log\lambda}.$$

Take $\lambda_0 = 2$.

OR

Q.3 **(a)** Use Golden Section Method with $n = 6$ to minimize the function **07**

$$f(\lambda) = 0.65 - \frac{0.75}{1 + \lambda^2} - 0.65\,\lambda\,tan^{-1}\frac{1}{\lambda}$$

in the interval $[0, 3]$.

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with the starting point $X_1 = [0\ 0]^T$ using Newton's method. **07**

Q.4	**(a)**	Explain the method of unrestricted search with fixed step size. What is its demerit? Explain also the exhaustive search method.	**07**
	(b)	Write the algorithm of Hooke and Jeeves'Method.	**07**

OR

Q.4	**(a)**	Use Interval Halving Method to minimize the function $$f(\lambda) = \lambda^5 - 5\lambda^3 - 20\lambda + 5$$ with $n = 7$ in the interval $(0, 5)$.	**07**
	(b)	Write the general form of an optimization problem. Write the classification of optimization problem.	**07**
Q.5	**(a)**	Use Cauchy'ssteepest descent method to minimize $$f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$$ from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform only two iterations.	**07**
	(b)	What are the transformation techniques? How they are useful in optimization? List some transformations to convert a constrained optimization problem into an unconstrained one?	**07**

OR

Q.5	**(a)**	Write the algorithm of Univariate method.	**07**
	(b)	Write a short note: Penalty function methods.	**07**

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