Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER–I(New course)• EXAMINATION – WINTER- 2015

Subject Code: 2710710 **Date: 31/12/2015**

Subject Name: Applied Linear Algebra

Time:2:30 pm to 5:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Determine the dimension of and a basis for the solution space of the system **07**

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

(b) (i) Is the vector $(3, -1, 0, -1)$ in the subspace of R^4 spanned by the vectors $(2, -1, 3, 2)$, $(-1, 1, 1, -3)$ and $(1, 1, 9, -5)$? **03**

(ii) Check whether the following set of vectors in P_2 are linearly independent : **04**

$p_1 = 1 - x, \quad p_2 = 5 + 3x - 2x^2, \quad p_3 = 1 + 3x - x^2$

Q.2 **(a)** Let F be a subfield of the complex numbers and let T be the function from F^3 into F^3 defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3,\ 2x_1 + x_2,\ -x_1 - 2x_2 + 2x_3)$ **07**

(i) Verify that T is a linear transformation.

(ii) If (a, b, c) is a vector in F^3, what are the conditions on a, b and c that the vector be in the range of T? What is the rank of T?

(iii) What are the conditions on a, b and c that (a, b, c) be in the null space of T? What is the nullity of T?

(b) (i) Let T and U be the linear operators on R^2 defined by $T(x_1, x_2) = (x_2, x_1)$ and $U(x_1, x_2) = (x_1, 0)$. Find the transformations $(U + T)$, UT and U^2. Also describe them geometrically. **03**

(ii) Let V be the set of pairs (x, y) of real numbers and let F be the field of real numbers. Define $(x, y) + (x_1, y_1) = (x + x_1, 0)$ **04**

$$c(x, y) = (cx, 0)$$

Is V, with these operations, a vector space?

OR

(b) State and prove rank – nullity theorem. **07**

Q.3 **(a)** Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix **07**

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Prove that T is diagonalizable by exhibiting a basis for R^3, each vector of which is a characteristic vector of T.

(b) Let L be a linear transformation on R^4 defined by **07**

$L[x,\ y,\ z,\ u] = [x+y,\ x-y,\ z+2u,\ 2z-u]$

(i) Show that the subspace S generated by $\{(a, b, 0, 0): a,b \in R\}$ and the subspace S' generated by $\{(0, 0, c, d): c,d \in R\}$ are L - invariant.

(ii) Determine the matrix of L with respect to the standard basis of R^4.

OR

Q.3 **(a)** Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix **07**

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

Find the minimal polynomial for the matrix.

(b) (i) Let $f(x) = x^2 + 3x + 2 + a$, where a is a real number, $d(x) = x + 1$ and $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Find the quotient $q(A)$ and the remainder $r(A)$ on dividing $f(A)$ by $d(A)$. **03**

(ii) Find A^{30} if $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ **04**

Q.4 **(a)** (i) Find $\|u\|$ and $d(u,v)$, where $u = (2,-1)$, $v = (-1,1)$ are in R^2 with the inner product defined as $\langle u, v\rangle = 2u_1v_1 - u_1v_2 - u_2v_1 + u_2v_2$. **03**

(ii) Show that $\langle a, b\rangle = a_1b_1 + a_2b_3 + a_3b_2 + a_4b_4$ is not an inner product on M_{22}, where $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$, $B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$ **04**

(b) Apply the Gram–Schmidt process to the vectors $u_1 = (1, 0, 1)$, $u_2 = (-1, 1, 0)$, $u_3 = (-3, 2, 0)$, to obtain an orthonormal bases for R^3 with the standard inner product. **07**

OR

Q.4 **(a)** Let S_1, S_2 and S_3 be the subspaces of R^3 generated by **07**

$S_1 = \{(a,b,c) / a+2b-c = 0\}$, $S_2 = \{(a,b,c) / b = c\}$ and

$S_3 = \{(a,b,c) / a-2c = 0, b+4c = 0\}$

Determine (i) basis for $S_2 \cap S_3$ and (ii) the dimension of $S_1 + S_2$

(b) Determine Smith canonical form of the following λ - matrix : **07**

$$A(\lambda) = \begin{bmatrix} \lambda^2 & \lambda^2 - 2\lambda & 8\lambda \\ \lambda & -\lambda & -2\lambda \\ \lambda^2 & \lambda^2 - 2\lambda & \lambda^2 + 7\lambda \end{bmatrix}$$

Q.5 (a) (i) Show that $A=\frac{1}{2}\begin{bmatrix}\sqrt{2} & -i\sqrt{2} & 0\\ i\sqrt{2} & -\sqrt{2} & 0\\ 0 & 0 & 2\end{bmatrix}$ is unitary. **03**

(ii) Let the vector space be C^2, with the standard inner product. Let T be the linear operator defined by $T\varepsilon_1=(1,-2), T\varepsilon_2=(i,-1)$. If $\alpha=(x_1,x_2)$, find $T^*\alpha$. **04**

(b) Solve the following system of linear equations by Cholesky's method: **07**

$$\begin{aligned} 4x + 2y + 14z &= 14\\ 2x + 17y - 5z &= -101\\ 14x - 5y + 83z &= 155 \end{aligned}$$

OR

Q.5 (a) Obtain the least squares solution of $AX=b$ and also the orthogonal projection of b on the column space of A where **07**

$$A=\begin{bmatrix}1 & 2\\ -1 & -5\\ 1 & 5\\ -1 & -2\end{bmatrix},\ b=\begin{bmatrix}6\\ 12\\ -6\\ 6\end{bmatrix}$$

(b) Using Jacobi's method, compute all the eigen values and the corresponding eigen vectors of the matrix **07**

$$A=\begin{bmatrix}2 & \sqrt{2} & 4\\ \sqrt{2} & 6 & \sqrt{2}\\ 4 & \sqrt{2} & 2\end{bmatrix}$$

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