Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER–I(New course)• EXAMINATION – WINTER- 2015

Subject Code: 2714601 **Date: 01/01/2016**

Subject Name: STATISTICS FOR ENGINEERS

Time:2:30 pm to 5:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Students are permitted to use statistical Tables**

Q.1		i.	If an experiment has the three possible and mutually exclusive outcomes A, B, and C, check whether the given assignment of probabilities is permissible. P(A) = 0.57, P(B) = 0.24, P(C) = 0.19	**02**
		ii.	Given that $f(x) = \frac{k}{2^x}$ is a probability distribution for a random variable that can take on the values $x = 0, 1, 2, 3$ and 4, find k.	**02**
		iii.	Use Table 1 to find $\sum_{k=4}^{10} b(k; 10, 0.35)$	**02**
		iv.	Use Table 2 to find f(9; 12)	**02**
		v.	For the uniform distribution $f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & for\ \alpha < x < \beta \\ 0, & elsewhere \end{cases}$ find mean and variance.	**02**
		vi.	Find $z_{0.01}$	**02**
		vii.	If a random variable has a normal distribution, what is the probability that it will take on a value within 3 standard deviation of the mean?	**02**
Q.2	**(a)**	i.	An oil company is bidding for the rights to drill a well in field A and a well in field B. The probability it will drill a well in field A is 0.40. If it does, the probability the well will be successful is 0.45. The probability it will drill a well in field B is 0.30. If it does, the probability the well will be successful is 0.55. Calculate each of the following probabilities: (a) probability of a successful well in field A, (b) probability of a successful well in field B, (c) probability of both a successful well in field A and a successful well in field B.	**03**
		ii.	A boiler containing eight welds is manufactured in a small shop. When the boiler is completed, each weld is checked by an inspector. If more than one weld is defective on a single boiler, the person who made that boiler is reported to the foreman. If 5.0% of all welds made by Kapil are defective, what percentage of all boilers made by him will have more than one defective weld?	**04**

(b) i. The number of meteors found by a radar system in any 30-second interval under specified conditions averages 1.80. Assume the meteors appear randomly and independently. (a) What is the probability that no meteors are found in a one-minute interval? (b) What is the probability of observing at least five but not more than eight meteors in two minutes of observation? **04**

ii. The waiting time X (in minutes) of a customer waiting to be served at a ticket counter has a density function $f_X(x) = \begin{cases} 2e^{-2x}; & x \geq 0 \\ 0, & elsewhere \end{cases}$ Determine the average waiting time. **03**

OR

(b) Prove that when n is large and p is small, the limiting form of the binomial distribution is the Poisson distribution. **07**

Q.3 (a) i. Let the phase error in a tracking device have probability density $f(x) = \begin{cases} cosx; & 0 < x < \frac{\pi}{2} \\ 0; & elsewhere \end{cases}$ Find the probability that the phase error is (a) between 0 and $\frac{\pi}{4}$; (b) greater than $\frac{\pi}{3}$. **04**

ii. The geometric distribution is $g(x;p) = p(1-p)^{x-1}$ for $x = 1, 2, 3, \ldots$. If the probability is 0.05 that a certain kind of measuring device will show excessive drift, what is the probability that the sixth measuring device tested will be the first to show excessive drift? **03**

(b) The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with μ = 12.9 minutes and σ = 2.0 minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take (a) at least 11.5 minutes; (b) anywhere from 11.0 to 14.8 minutes? **07**

OR

Q.3 (a) i. In a certain city, the daily consumption of electric power (in millions of Kilowatt-hours) is a random variable having the probability density $f(x) = \begin{cases} \frac{1}{9}xe^{-x/3}; & x > 0 \\ 0; & x \leq 0 \end{cases}$ If the city's power plant has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day? **04**

ii. Prove that the identity $\sigma^2 = \mu_2' - \mu_2$ **03**

(b) The Weibull distribution is **07**

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}} & for\ x > 0,\ \alpha > 0, \beta > 0 \\ 0 & elsewhere \end{cases}$$

Suppose that the lifetime of a certain kind of an emergency backup battery (in hours) is a random variable X having the Weibull distribution with α = 0.1 and β = 0.5. Find (a) the mean lifetime of these batteries; (b) the probability that such a battery will last more than 300 hours.

Q.4 (a) Let X_1 and X_2 have the joint probability distribution in the table below. **07**

Joint Probability Distribution $f(x_1, x_2)$ of X_1 and X_2				
		x_1		
x_2		0	1	2
	0	0.1	0.4	0.1
	1	0.2	0.2	0

(a) Find $P(X_1 + X_2 > 1)$ (b) Find the probability distribution $f_1(x_1) = P(X_1 = x_1)$ of the individual random variable X_1.

(b) A particular production process used to manufacture ferrite magnets used to operate reed switches in electronic meters is known to give 10% defective magnets on average. If 200 magnets are randomly selected, what is the probability that the number of defective magnets is between 24 and 30? Use a normal approximation to binomial probabilities. **07**

OR

Q.4 **(a)** Define the moment generating function $M(t)$ of a discrete random variable X. Let X have the Poisson distribution with probability distribution **07**

$f(x) = \frac{\lambda^x}{x!} e^{-\lambda}; \quad x = 0, 1, 2, \ldots$ Show that

(a) $M(t) = e^{\lambda(e^t - 1)}$ for all t (b) $E(X) = \lambda, \qquad var(X) = \lambda$

(b) i. A random sample of size n = 100 is taken from a population with σ = 5.1. Given that the sample mean is $\bar{x}$ = 21.6, construct a 95% confidence interval for the population mean μ. **04**

ii. The mean weight loss of n = 16 grinding balls after a certain length of time in mill slurry is 3.42 grams with a standard deviation of 0.68 gram. Construct a 99% confidence interval for the true mean weight loss of such grinding balls under the stated conditions. **03**

Q.5 **(a)** Two tests are made for the compressive strength of each of 6 samples of poured concrete. The force required to crumble each of 12 cylindrical specimens, measured in kg, is as follows: **07**

SAMPLE

	A	B	C	D	E	F
Test-1	110	125	98	95	104	115
Test-2	105	130	107	91	96	121

Test at 0.05 level of significant whether these samples differ in compressive strength.

(b) An experiment was performed to judge the effect of 4 different fuels along with the effects of 2 different launchers on the range of a certain rocket. The data (in nautical miles) is given in following table: **07**

	Fuel I	Fuel II	Fuel III	Fuel IV
Launcher X	62.5	49.3	33.8	43.6
Launcher Y	40.4	39.7	47.4	59.8

Considering fuels as treatments and launchers as blocks prepare the ANOVA table and test at $\alpha = 0.05$ significance level whether the differences among the means corresponding to fuels are significant. Also test whether the blocking has been effective.

OR

Q.5 **(a)** To compare the effectiveness of three methods of teaching the programming compute method A, B, and C –Random sample of size 4 are taken from large group and the following are the scores which they obtained. **07**

Method A	Method B	Method C
73	91	72
77	81	77
67	87	76
71	85	79

Test at the level of confidence $\alpha = 0.05$ whether the differences among the means obtained for these three methods are significant?

(b) An experiment was designed to study the performance of 4 detergents for cleaning fuel injectors. The following "cleanliness" reading were obtained for specially designed equipment for 12 tanks of gas distributed over 3 different models of engines: **07**

	Engine 1	Engine 2	Engine 3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	42	37	49

Consider the detergents as treatments and engines as blocking, prepare ANOVA table and perform F test for variances due to the treatments and the blocking. Write your conclusions.

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