Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E. SEMESTER I (old course)–EXAMINATION (Remedial) – WINTER 2015

Subject code: 710401 **Date: 08/12/2015**

Subject Name: Statistical Signal Analysis

Time: 10:30 AM to 1:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** State and prove the Central Limit Theorem and explain its importance in Communication Engineering. **07**

(b) Give the PDF for Gaussian distributed random variable and show that Gaussian PDF integrates to one. **04**

(c) Give definition of characteristic function of a Random variable and find the characteristic function of an exponentially distributed random variable. **03**

Q.2 **(a)** Describe the importance of probability theory in Communication Engineering. **07**

(b) Find the pdf of **07**

(i) Y = aX+b where x is a random variable with Gaussian pdf with mean m and standard deviation ů

(ii) Y= cos (X) where X is uniformly distributed in the interval [0,2´]

OR

(b) (i) A random variable X has cdf **07**

$F_x(x)$ = 0 xÒ-´/2

= c(1+sin(x)) -´/2 ÒxÒ´/2

= 1 ´/2 Òx

(a) Find c (b) find the pdf of x

Q.3 **(a)** (i) The probability of bit error in communication line is 10^{-3} Find the probability that a block of 1000 bits has five or more errors **07**

(ii) Show that the pdf of a Gamma variable given below integrates to 1

$$f_x(x) = \frac{\lambda(\lambda x)^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$$

(b) Find the mean and variance of a uniform random variable which is distributed in the interval [a,b] **07**

OR

Q.3 **(a)** State and Explain the Markov and Chebychev inequality **07**

(b) Find the normalization constant c and the marginal pdfȏs of the joint pdf given by **07**

fx,y (x,y) = ce ^{ïx} e ^{ïy} 0 ÒyÒx< Ð

= 0 elsewhere

Q.4 **(a)** (i) Let X be the input to a communication channel and Y be the output. The input is + 1 or -1 volt with equal probability. The output of the channel is input plus noise voltage N which is uniformly distributed in the interval -2 volts to + 2 volts .Find $P[X=+1, Y \leq 0]$ **07**

(ii) A computer system receives messages over three communication lines .Let Xj be the messages received on line j in one hour. Suppose that the joint pmf of X_1 , X_2,X_3 is given by

$p_{X1,X2,X3}(x_1,x_2,x_3) = (1-a_1)(1-a_2)(-a_3)\ a_1^{x1}\ a_2^{x2}\ a_3^{x3}$

$x_1,x_2,x_3 \geq 0$. Find the marginal pmf's $p_{X1,X2}(x_1,x_2)$ and $p_{X1}(x_1)$.

(b) Give the definition of Covariance and Correlation coefficient of two random variable. Prove giving suitable examples the statement that "If X and Y are independent they are uncorrelated but the reverse is not always true". **07**

OR

Q.4 **(a)** If Z = X+ Y where X and Y are random variables .Find E[Z] and variance of Z. **07**

Q.4 **(b)** Explain (i) Sure Convergence (ii) Almost sure convergence (iii) Mean convergence and Cauchy Criteria. **07**

Q.5 **(a)** If $X(t) = A \cos 2\pi t$, Where A is some random variable find the (i) Mean (ii) Auto correlation and (iii) Autocovariance of the random process X(t). **07**

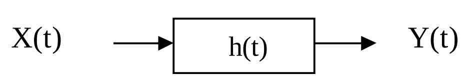
(b) Find the Power spectral density for a polar binary random signal where 1 is transmitted by a pulse p(t) and zero is transmitted by a pulse – p(t) .The digits 1 and 0 are equally likely one digit is transmitted every T_b second. Each digit is independent of other digits. **07**

OR

Q.5 **(a)** Explain the terms (i) Stationary Random Process (ii) Widesense stationary Random process (iii) Cyclostationary Random process. **07**

(b) Show that the power spectral density of a linear time invariant system shown below is given by **07**

$S_y(f) = |H(f)|^2 S_x(f)$



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