Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E. SEMESTER I (old course)–EXAMINATION (Remedial) – WINTER 2015

Subject code: 710901N **Date: 08/12/2015**

Subject Name: Theory of Elasticity

Time: 10:30 AM to 1:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** For the plane state of stress derive; **07**

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}{}^2 \right]^{1/2}$$

$$\tau_{\max} = \left[\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}{}^2 \right]^{1/2}$$

(b) (i) Displacement field for a body is given by u = $(x^2+ y)$i + (3 + z)j + $(x^2 + 2y)$k. Find the deformed position of the point originally at (3, 1, -2). **03**

(ii) Explain with a neat schematic sketch the interpretation of shear strain ɔ$_{xy}$ in xy plane for two fibers PQ and PR mutually perpendicular to each other and parallel to x and y axes. **04**

Q.2 **(a)** For the displacement field u = $[y^2$i + 3yzj + $(4+6x^2)$k$]10^{-2}$; determine the rectangular strain components at point P(1, 0, 2). **07**

(b) ñThe three values of principal stresses are always real and not imaginaryò. Comment on the degree of the validity of a given statement with a mathematical proof. **07**

OR

(b) For the given state of stress, determine the principal stresses and their directions. **07**

$$[\tau_{ij}] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Q.3 **(a)** Derive following differential equations of equilibrium; **07**

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \gamma_x = 0$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \gamma_y = 0$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + \gamma_z = 0$$

Where, ů$_x$, ů$_y$, ů$_z$, Ǘ$_{xy}$, Ǘ$_{yz}$, Ǘ$_{zx}$ are rectangular stress components and ɔ$_x$, ɔ$_y$, ɔ$_z$ are body force components per unit volume in x, y and z directions.

(b) Show how the strain matrix can be divided into hydrostatic stress and deviatoric stress. State the significance of deviatoric component of stress as regards to plastic deformation. **07**

OR

Q.3 **(a)** At point P in a body, σ_x = 10,000 N/cm^2, σ_y = -5,000 N/cm^2, σ_z = -5,000 N/cm^2, τ_{xy} = τ_{yz} = τ_{zx} = 10,000 N/cm^2. Determine the normal and shearing stresses on a plane that is equally inclined to all the three axes. **07**

(b) For the given state of strain, determine the principal strains and the direction of maximum and minimum principal strains. **07**

$$[\varepsilon_{ij}] = \begin{bmatrix} 0.02 & -0.04 & 0 \\ -0.04 & 0.06 & -0.02 \\ 0 & -0.02 & 0 \end{bmatrix}$$

Q.4 **(a)** Write a Hooke's law and extend this concept to derive generalized Hooke's law for relating six strain components with stress components assuming linear variation of stress with strain for homogeneous material. Using this law derive a stress strain relationship for linear, elastic, homogeneous and isotropic material. **07**

(b) Define bulk Modulus K and express it in terms of the Lame's coefficient λ and ε. For steel having E = 207 x 10^6 kPa and ν = 0.3 calculate λ, ε and K. **07**

OR

Q.4 **(a)** A rubber cube as shown in the figure 1 is inserted in the cavity of same form and size in a steel block and top of the cube is pressed by a steel block with a pressure of p pascals. Considering the steel to be absolutely hard and assuming that there is no friction between steel and rubber, find (i) the pressure of rubber against box walls and (ii) the extremum shear stresses in rubber. **07**

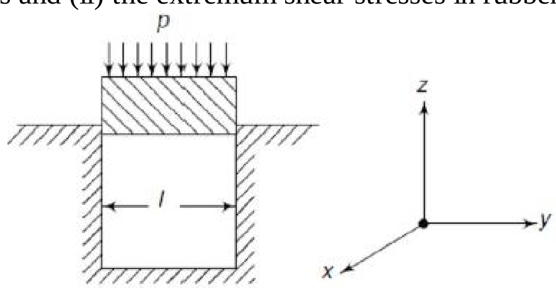

Figure 1

(b) Draw and comment on the nature of the Mohr's circle diagram for the following cases where in the three principal stresses σ_1, σ_2 and σ_3 are given as: (i) unequal (ii) equal (iii) any two of them are equal. **07**

Q.5 **(a)** In reference to Menabrea's theorem prove that forces developed in a redundant framework are such that the total elastic strain energy is a minimum. **07**

(b) (i) Define axisymmetric problems giving suitable example. **02**

(ii) Define circumferential strain, radial strain and axial strain with the help of necessary sketch and expressions. **05**

OR

Q.5 **(a)** Derive Castigliano's first theorem and show that the partial differential coefficient of the strain energy function with respect to force Fr, gives displacement corresponding with Fr. **07**

(b) For the thermoelastic problem, if the body is subjected to a uniform temperature rise T = $T_0(t)$ and if the body is prevented from having any displacements, then show that; **07**

$$\sigma_x = \sigma_y = \sigma_z = -\frac{E\alpha}{1-2\nu} T_0$$

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