

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC - SEMESTER-II EXAMINATION – SUMMER 2016

Subject Code:X20001**Date:27/05/2016****Subject Name:MATHEMATICS-2****Time:10:30 AM to 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1	(a) (1) Prove that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$.	02
	(2) Find $1 * 1$.	02
	(3) Define: Beta function. Prove that $\beta(m, n) = \beta(n, m)$	03
(b)	(1) State and prove First shifting property in Laplace transform.	02
	(2) Find $L(t \sin 4t)$.	02
	(3) Evaluate: $\int_0^1 x^4 (1 - \sqrt{x})^5 dx$.	03
Q.2	(a) (1) State Convolution theorem and hence find $L^{-1}\left(\frac{1}{s^2 - s - 12}\right)$.	03
	(2) Evaluate $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.	02
	(3) Find $L[e^{-3t}(2\cos 5t - 3\sin 5t)]$.	02
(b)	(1) Using Laplace transform solve, $y'' + 9y = 6$, $t \geq 0$, $y(0) = 2$, $y'(0) = 0$.	04
	(2) Find the Laplace transform of	03
	$f(t) = \begin{cases} 0, & 0 \leq t \leq 2 \\ 3, & t \geq 2 \end{cases}$	
	OR	
(b)	(1) Using Laplace transform solve, $y'' + 4y = 0$, $t \geq 0$, $y(0) = 1$, $y'(0) = 6$.	04
	(2) Evaluate $L^{-1}\left[-\frac{s+10}{s^2 - s - 2}\right]$.	03
Q.3	(a) Find the Fourier series for $f(x) = x^2$, $x \in (-\pi, \pi)$.	05
	Hence, deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$	
(b)	Find the Fourier series for $f(x) = x$, $x \in (0, 2\pi)$.	05
(c)	Find the Half range Fourier series for $f(x) = x - x^2$, $x \in (0, \pi)$.	04
	OR	
Q.3	(a) Find the Fourier series for $f(x) = 4 - x^2$, $x \in (0, 2)$.	05
(b)	Using Fourier integral, show that	05
	$\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda d\lambda = \begin{cases} \frac{\pi}{2} & \text{if } x \in (0, \pi) \\ 0 & \text{if } x \geq \pi \end{cases}$	
(c)	Find the Fourier series for $f(x) = x^3$, $x \in (-\pi, \pi)$.	04

- Q.4** (a) Solve : $(1+2x)^2 y'' - 6(1+2x)y' + 16y = 8(1+2x)^2$. 05
 (b) Solve by method of variation of parameter $D^2 y + y = \sec x$. 05

- (c) Solve I.V.P. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$, $y(0) = 4$, $y'(0) = 5$. 04

OR

- Q.4** (a) Solve using Cauchy-Euler equation, $x^2 y'' - 4xy' + 6y = x$. 05
 (b) Solve by method of variation of parameter $y'' + y = \cos ex$. 05
- (c) Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$. 04

- Q.5** (a) State Convolution theorem, find z-transform of 05

$$f_1(k) = \{2, 3, 4\}, f_2(k) = \{-1, 2, 3\}.$$

- (b) Solve: $y^2 zp + x^2 zq = xy^2$. 05

- (c) Solve by method of separation of variables, 04

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}, \text{ where } u(0, y) = 8e^{-3y}.$$

OR

- Q.5** (a) State change of scale property. Find the z-transform of $a^k \sin \beta k$, $k \geq 0$. 05

- (b) Solve: $q = 3p^2$. 05

- (c) Solve by method of separation of variables, 04

$$x \frac{\partial u}{\partial x} - 2y \frac{\partial u}{\partial y} = 0.$$
