Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**PDDC - SEMESTER–VI. EXAMINATION – SUMMER 2016**

**Subject Code:X61103** **Date:06/05/2016**

**Subject Name:Digital Signal Processing**

**Time:10:30 AM TO 01:00 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Explain the sampling of a signal $x(t)=\cos(2\pi t)$ at two different sampling rates 2Hz and 4Hz, respectively. Sketch the sampled signal and the spectrum, respectively in each of the two cases. **07**

**(b)** What is the function of interpolation filter? Specify and sketch the frequency response of an ideal interpolation filter for a case where it is required to up-sample a discrete time signal x[n] by a factor of two. **07**

**Q.2** **(a)** Explain the two conditions of an LTI system: Linear and Time-invariant. State which of these conditions are satisfied for the systems: **07**

(i) $y[n]=x[n-1]\, x[n+1]/x[n]$, if x[n] is not zero, y[n]=0 if x[n]=0; and

(ii) $v[n]=(2n+1)x[n]$.

**(b)** Derive an inverse system of the system, $y[n]= x[n] - x[n-1]$ for $n\geq 0$. **07**

**OR**

**(b)** If the output y[n] of a system is the convolution sum of the input x[n] and the impulse response h[n], then derive y[n] for the input $x[n]=\{e^{j\lambda n} \mid \lambda \in (-\pi, \pi)\}$ in the form $y[n]= f(\lambda)\, x[n]$. What is $f(\lambda)$ here? **07**

**Q.3** **(a)** State the necessary and sufficient conditions for BIBO stability of any LTI system with respect to the impulse response h[n]. Also give the proof of the statements. **07**

**(b)** Give the equations for computing N-point DFT (Discrete Fourier Transform) and IDFT (Inverse Discrete Fourier Transform). State and explain the symmetry conditions and the periodic properties of the DFT. **07**

**OR**

**Q.3** **(a)** Any sequence x[n] can be shown as a weighted superposition of the time-shifts of the unit impulse function $\delta[n]$ as, $x[n] = \sum_{k\in Z} x[k]\, \delta[n-k]$. If h[n] is the impulse response of an LTI system then give the step-by-step derivation of the output from the above given superposition form of input x[n] using LTI conditions. **07**

**(b)** If the output y[n] of a system is the N-point cyclic convolution sum of the input x[n] and the impulse response h[n], then derive the corresponding relationship in the spectral domain. Assume y[n], x[n], and h[n] are N-point periodic. **07**

**Q.4** **(a)** Give a general z-transform equation of a second order ARMA (Auto-Recursive Moving Average) system. Give Direct Form – I, II and Transposed Form – I, II realizations of the system. **07**

**(b)** Derive the z-transform and the corresponding ROC (Region of Convergence) for the system $y[n] = x[n] + 0.1\, y[n-1] + 0.12\, y[n-2]$ for $n\geq 0$. State whether the system is stable or not and why? **07**

**OR**

**Q.4** **(a)** Find the difference equation of the following system **07**

$H(z)= (1+(3/4)z^{-1}+(9/16)z^{-2}) /(1+(2/5)z^{-1}+(4/25)z^{-2})$, ROC: $(2/5)<|z|$.

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| | **(b)** | Derive the impulse response of an ideal LPF (Low Pass Filter) with cut-off frequency $\omega_c$. | **07** |
| **Q.5** | **(a)** | Write a short note on Windowing method for design of FIR filters. | **07** |
| | **(b)** | Write a short note on Mapping methods for design of IIR filters. | **07** |

**OR**

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|---|---|---|---|
| **Q.5** | **(a)** | Sketch the signal flaw graph for DIF (Decimation-in-Frequency) FFT algorithm. | **07** |
| | **(b)** | Explain MAC unit and pipe-lining in DSP. | **07** |

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