Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER–II EXAMINATION – WINTER 2015

Subject Code: X20001 **Date:19/12/2015**

Subject Name: Mathematics-II

Time: 02:30pm to 05:30pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Solve : $y''' + 2y'' + y' = \cos^2 x$ **07**

(b) Solve the equation $y'' + y = x\sin x$ using the method of variation of parameters. **07**

Q.2 **(a)** Solve: $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0 \,\&\, z = 0$, when y is an odd multiple of $\frac{\pi}{2}$. **07**

(b) (i) Prove that $\overline{/(n+1)} = \text{n!}$, where n is positive integer. **03**

(ii) Prove that $\beta(m,n) = \beta(n,m)$. **04**

OR

(b) Evaluate : $\int_0^1 \frac{x^2}{(1-x^4)^{\frac{1}{2}}} dx \cdot \int_0^1 \frac{x^2}{(1+x^4)^{\frac{1}{2}}} dx$ **07**

Q.3 **(a)** Find the Fourier series of $f(x) = e^{-x}, -l < x < l.$ **07**

(b) Express $\sin x$ as a cosine series in $0 < x < \pi$. **07**

OR

Q.3 **(a)** Find the Fourier series for the function given by $f(x) = \begin{cases} \pi x, 0 \le x \le 1 \\ \pi(2-x), 1 \le x \le 2. \end{cases}$ **07**

(b) Find the Fourier series for x^2 in the interval $-\pi < x < \pi$. **07**

Q.4 **(a)** Prove that $L(sinat) = \frac{a}{s^2 + a^2}$ and $L(t^n) = \frac{n!}{s^{n+1}}$. **07**

(b) Using convolution theorem , find $L^{-1}\left(\frac{1}{(s^2+a^2)^2}\right)$. **07**

OR

Q.4 **(a)** (i) Find $L\left(\frac{1-\cos t}{t}\right)$. **03**

(ii) Find the Laplace transform of $f(\text{t}) = \begin{cases} 0, 0 < t < \pi \\ \text{sint}, t > \pi \end{cases}$. **04**

(b) Find $L^{-1}\left(\frac{(5s+3)}{(s^2+2s+5)(\text{s}-1)}\right)$. **07**

Q.5 **(a)** The tightly stretched string with fixed and point $x=0 \& x=l$ is initially in a position given by $u(x,0)=u_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it released from rest from this position, find the displacement $u(x,\mathrm{t})$ an any time & at any distance from the end $x=0$. **07**

(b) Find $Z^{-1}\left[\frac{z-6}{z^2+5z+6}\right]$. **07**

OR

Q.5 **(a)** Solve: $2\frac{\partial u}{\partial x}=\frac{\partial u}{\partial t}+u$, subject to the condition $u(x,0)=4e^{-3x}$ **07**

(b) (i) Solve: $\frac{\beta(m,n+1)}{n}=\frac{\beta(\mathrm{m}+1,\mathrm{n})}{m}=\frac{\beta(\mathrm{m},\mathrm{n})}{m+n}$ **04**

(ii) Solve the IVP : $y''+y'-2y=0$ where $y(0)=4 \& y'(0)=-5$. **03**

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